SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2019

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: TECHNICAL MATHEMATICS

TEACHER NOTES

SESSIONS 1 TO 6

SESSION NO: 1

TOPIC: EUCLIDEAN GEOMETRY THEORY TO THE END OF GRADE 11

LESSON OVERVIEW

1.	Introduction session –See **Learner note**	5 minutes
2.	Work through **Theorems relating to straight lines; Theorems relating to parallel lines** and **Theorems relating to triangles** with learners. See **SECTION B: NOTES ON CONTENT**	10 minutes
3.	Learners do Questions 1 and 2	20 minutes
4.	Discuss solutions	15 minutes
5.	Work through **Theorems relating to circles** with learners. See **SECTION B: NOTES ON CONTENT**	10 minutes
6.	Learners do Questions 3 and 4	20 minutes
7.	Discuss solutions	10 minutes

TEACHER NOTE:

1. The aims of this session are
   - to revise theory that is used in Grade 11 Euclidean geometry and to highlight theory where learners often get confused
   - to revise methods of proof i.e. logical deduction and congruency proofs
   - To remind learners where reasons should be given: with statements and also when concluding that lines are parallel or triangles are congruent.
2. Circle diagrams are used as in the next sessions learners will answer the actual exam questions based on the diagrams from the first session..
3. The abbreviations used for the statements are from the CAPS Mathematics GR 12 Exam Guidelines 2017.
4. The timing here is less than a mark a minute as most of the answers are short.
5. The Homework questions allow for a mark a minute.
6. The mark allocations in this section are to show the learners what they must write in their answers. A proper exam type question may not allocate as many marks to an answer.

SESSION NO: 2

TOPIC: EUCLIDEAN GEOMETRY EXAM TYPE QUESTIONS 1: (GRADE 11)
LESSON OVERVIEW

1. Introduction session – Answer queries on Homework;
   Discuss the first sentence of the **Learner Note** and tell learners that
   . Session 1 prepared them for this. 5 minutes
2. Work through to $A\hat{B}E = \hat{A}_1$ of Question 1.1 with the learners.
   Discuss the HINTS mentioned in the rest of the **Learner Note** as you do so.
   You may also want to go through the important parts that should be underlined in all the questions. 20 minutes
3. Typical exam questions to be done by learners in the lesson
   Question 1 16 minutes
   Question 2: 11 minutes
   Question 3: 5 minutes
   Question 4: 6 minutes
3. Discussion of solutions: 27 minutes

TEACHER NOTE:
The solutions to these questions are more explicit than the memorandum for the exemplar. The reason for this is to assist learners in using a logical approach when doing Euclidean geometry.
Developing logic:

- Aim to include something about what you have to find in the first statement of your solution.
- Repeat something from the first statement in the second statement.
- Try to make a conclusion after two (at most three) statements.

Learners should try to answer Questions 2 to 4 and the homework questions under exam conditions.

SESSION NO: 3

TOPIC: EUCLIDEAN GEOMETRY EXAM TYPE QUESTIONS 2: (GRADE 11)

LESSON OVERVIEW

1. Introduction session – Suggest to learners that these questions be answered in exam conditions. 5 minutes
2. Typical exam questions to be done by learners in the lesson (~ 30 minutes):
   - Question 1 6 minutes
   - Question 2: 6 minutes
   - Question 3 7 minutes
   - Question 4: 4 minutes
3. Discussion of solutions: 62 minutes

TEACHER NOTE:

These questions have been taken from Mathematics past papers and have been adapted so that the work is more straight forward for the Technical maths learner.

Remind learners to

- fill in the given information on the diagram
- highlight centres, parallel lines and tangents on the diagram. Mark radii equal.
- keep re-reading the question
- to fill in what they have found out on their diagram
- to use previous answers to help them answer the current sub-question

SESSION NO: 4

TOPIC: COMPLEX NUMBERS OVERVIEW

LESSON OVERVIEW

1. Introduction session –See **Learner note** 5 minutes
2. Work through **SECTION B: NOTES ON CONTENT** Discuss 7. Solving Complex equations **Worked Example 2** in Session No: 5. 20 minutes
3. Learners do Questions 1 to 9 50 minutes
4. Discuss solutions 15 minutes

TEACHER NOTE:

1. Revise laws of surds including rationalising denominators when discussing Formats of complex numbers.

The following notes are from the Technical Mathematics Grade 11Learner's Book

Worked example 1.13 Simplify: $\sqrt{27} + \sqrt{8} + \sqrt{75}$ **Solution** $\sqrt{27} + \sqrt{8} + \sqrt{75}$ $= \sqrt{3 \times 9} + \sqrt{2 \times 4} + \sqrt{3 \times 25}$ $= 3\sqrt{3} + 2\sqrt{2} + 5\sqrt{3}$ $= 8\sqrt{3} + 2\sqrt{2}$	Write surds as the product of a perfect square and a non-perfect square

Worked example 1.25

Simplify the expression $(3+\sqrt{2})(3-\sqrt{2})$

Solution

$(3+\sqrt{2})(3-\sqrt{2})$

$=(3)^2-(\sqrt{2})^2$

$=9-2$

$=7$

Worked example 1.26

Simplify $(3+\sqrt{2})^2$.

Solution

$(3+\sqrt{2})^2$

$=(3+\sqrt{2})(3+\sqrt{2})$

$=(3)^2+2\times 3\times\sqrt{2}+(\sqrt{2})^2$

$=9+6\sqrt{2}+2$

$=11+6\sqrt{2}$

Worked example 1.27

Simplify $(3-\sqrt{2})(1+\sqrt{3})$.

Solution

$(3-\sqrt{2})(1+\sqrt{3})$

$=3\times 1+3\sqrt{3}-1\sqrt{2}-\sqrt{2}.\sqrt{3}$

$=3+3\sqrt{3}-\sqrt{2}-\sqrt{6}$

Note:

$x^2-y^2=(x-y)(x+y)$ is useful in simplifying surds.

$(x+y)^2=x^2+2xy+y^2$

$(x-y)^2=x^2-2xy+y^2$

The following two examples on rationalising denominators are from Siyavula Mathematics Grade 11

In both cases the denominators become rational numbers

QUESTION

Rationalise the denominator:

$$\frac{5x-16}{\sqrt{x}}$$

SOLUTION

Step 1: Multiply the fraction by $\frac{\sqrt{x}}{\sqrt{x}}$

Notice that $\frac{\sqrt{x}}{\sqrt{x}} = 1$, so the value of the fraction has not been changed.

$$\frac{5x-16}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}(5x-16)}{\sqrt{x} \times \sqrt{x}}$$

Step 2: Simplify the denominator

$$= \frac{\sqrt{x}(5x-16)}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{x}(5x-16)}{x}$$

This is similar to $\frac{4-19i}{3i}$

QUESTION

Write the following with a rational denominator:

$$\frac{y-25}{\sqrt{y}+5}$$

SOLUTION

Step 1: Multiply the fraction by $\frac{\sqrt{y}-5}{\sqrt{y}-5}$

To eliminate the surd from the denominator, we must multiply the fraction by an expression that will result in a difference of two squares in the denominator.

$$\frac{y-25}{\sqrt{y}+5} \times \frac{\sqrt{y}-5}{\sqrt{y}-5}$$

Step 2: Simplify the denominator

$$= \frac{(y-25)(\sqrt{y}-5)}{(\sqrt{y}+5)(\sqrt{y}-5)}$$

$$= \frac{(y-25)(\sqrt{y}-5)}{(\sqrt{y})^2-25}$$

$$= \frac{(y-25)(\sqrt{y}-5)}{y-25}$$

This is similar to $\frac{4-19i}{3+i}$

2. After discussing “Using a calculator to convert between the rectangular and polar Forms” mention that exam questions will probably state “using a formula” if they do not want learners to use the calculator method. Remind learners that $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ and **not** $\theta = \tan^{-1}\left(\frac{a}{b}\right)$ and they must substitute positive values for a and b because putting in $180° \pm$ or $360° -$ will determine the angle sizes.
3. Remind learners when simplifying something like: $2\cos 40° + 2\sin 40°i$ to first calculate $2\cos 40°$ and then calculate $2\sin 40°$.
3. Applications are dealt with in Section 5.
4. The questions in Sessions 4 have been adapted from the LTSMs and do not include solving simultaneous equations – there are questions on solving simultaneous equations in Section 5.
5. Some timing in the questions are less than a mark a minute as most of the answers are short.

SESSION NO: 5

TOPIC TYPICAL EXAMINATION QUESTIONS ON COMPLEX NUMBERS

LESSON OVERVIEW

1. Introduction session – Answer queries on Homework;
   Discuss the first sentence of the **Learner Note** 5 minutes
2. Work through **SECTION B: NOTES ON CONTENT** and discuss
   7. Solving Complex equations **Worked Example 2** 10 minutes
3. Learners do Questions 1 to 6 Typical exam questions 30 minutes
4. Discussion of solutions: 45 minutes

TEACHER NOTE:
The questions in Sessions 6 have been adapted from revision questions in the LTSMs and do involve solving simultaneous equations as well as using complex numbers in electricity questions.

SESSION NO: 6

TOPIC: CIRCLES, ANGLES AND ANGULAR MOVEMENT

LESSON OVERVIEW

1. Introduction session –See **Learner note** 5 minutes
2. Work through **SECTION B: NOTES ON CONTENT** with learners 20 minutes
3. Learners do Questions 1 to 8 30 minutes
4. Discuss solutions 20 minutes
5. Allow learners to start the homework questions.
   They could start off by matching the classwork questions to the homework questions. 25 minutes

TEACHER NOTE:
Questions have been taken or adapted from the LTSMs or made up. Very few hints on which formula to use are given in the classwork as it is hoped that the learners use their notes to decide on the formula to use. The questions are based on the types of questions in the exemplar and November 2017 exams.
Question 6 is from the 2017 exemplar.
The memo for the homework solutions (the November 2017 exam) is as per the given marking memorandum – the only difference is that reasons for the ticks have been written in words so it is easier for the learners to mark their work.