SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2019

GRADE 12

SUBJECT: TECHNICAL MATHEMATICS

LEARNER SOLUTIONS

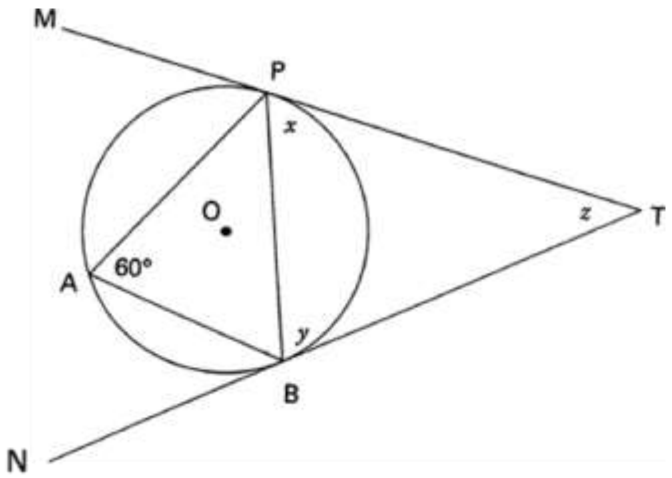
(Page 1 of 39)

SESSION 1

TOPIC: EUCLIDEAN GEOMETRY THEORY TO THE END OF GRADE 11

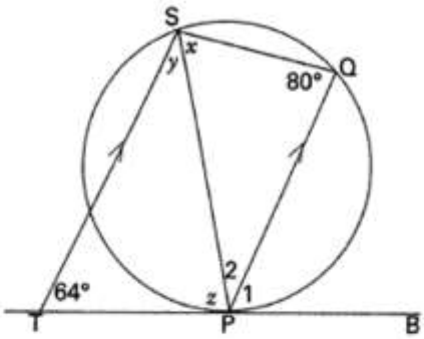
SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

QUESTION 1



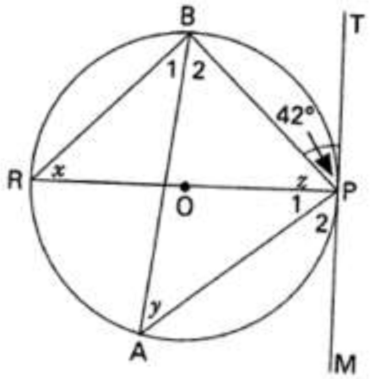
1.1	$x = 60°$ (tan chord theorem) $y = 60°$ (Tans from same pt) or (tan chord theorem) $60° + 60° + z = 180°$ (Int ∠s Δ PBT) $\therefore z = 60°$	✓ statement ✓reason ✓ statement ✓reason ✓ statement ✓reason ✓ angle size (7)
1.2	PT = TB = BP (sides opp equal ∠s)	✓ statement ✓reason (2)

QUESTION 2



2	$\hat{P}_1 = 64°$ (corr ∠s TS || PQ) $\hat{P}_1 = x$ (tan chord theorem) $\therefore x = 64°$ $y + 64° + 80° = 180°$ (co-int ∠s TS || PQ) $\therefore y = 36°$ $z = 80°$ (tan chord theorem)	✓ statement ✓reason ✓ statement ✓reason ✓ angle size ✓ statement ✓reason ✓ angle size ✓ statement ✓reason (10)

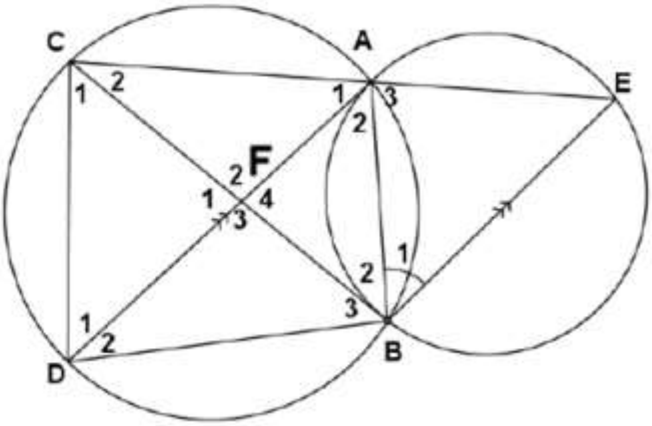
QUESTION 3



3.1 a)	$R\hat{P}M = 90°$ (tan ⊥ diameter)	✓ statement ✓reason (2)
3.1 b)	$R\hat{B}P = 90°$ (∠s in semi circle)	✓ statement ✓reason (2)
3.2	$x = 42°$ (tan chord theorem) $y = 42°$ tan chord theorem) or (∠s in the same seg) because $x = y$ $z + 42° = 90°$ (tan ⊥ diameter) $\therefore z = 48°$	✓ statement ✓reason ✓ statement ✓reason ✓ statement &reason ✓ angle size (6)

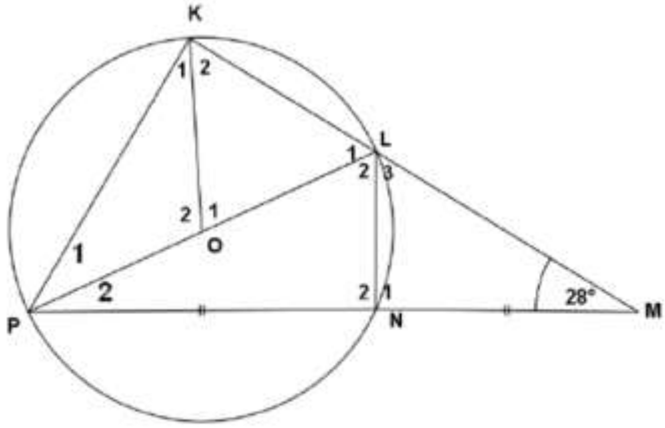
SECTION B: SOLUTIONS TO RECOGNISING THEORY IN TYPICAL EXAM DIAGRAMS

QUESTION 1



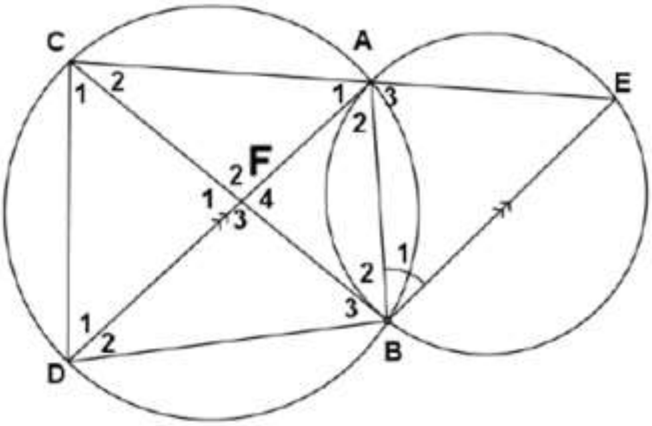
1.1	$\hat{A}_1 = \hat{E}$.... (corresp ∠s; DA||BE) $\hat{F}_2 = \hat{B}_1 + \hat{B}_2$ .... (corresp ∠s; DA||BE) $\hat{A}_2 = \hat{B}_1$.... (alt ∠s; DA||BE)	✓ statement ✓reason ✓ statement ✓reason ✓ statement ✓reason (6)
1.2	$\hat{D}_2 + \hat{B}_3 + \hat{B}_2 + \hat{B}_1 = 180°$ (co-int ∠s; DA||BE) $\hat{A}_2 + \hat{A}_3 + \hat{E} = 180°$ (co-int ∠s; DA||BE)	✓ statement ✓reason ✓ statement ✓reason (4)
1.3	$\hat{B}_1 + \hat{E} = \hat{A}_1 + \hat{A}_2$ (ext ∠ of Δ) can state which triangle namely: ΔABE	✓ $\hat{A}_1 + \hat{A}_2$ ✓ reason (2)

QUESTION 2



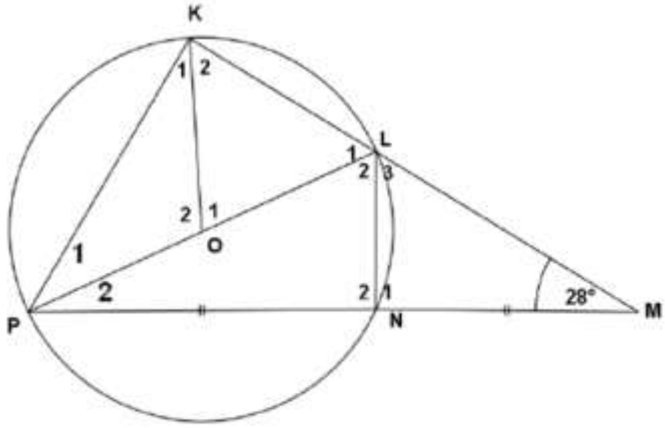
2.1	There are 3 radii coming off O so KO = PO = LO $\hat{K}_1 = \hat{P}_1$... $\hat{K}_2 = \hat{L}_1$... (∠s opp equal sides)	✓ statement ✓ statement ✓ reason (3)
2.2	In ΔLPN and ΔLMN PN = NM (given) LN = LN (common) $\hat{N}_1 = 90°$ (given) $\hat{N}_1 + \hat{N}_2 = 180°$ (∠s on a str line) $\therefore \hat{N}_1 = \hat{N}_2$ ∴ ΔLPN ≡ ΔLMN (SAS) $\therefore \hat{L}_2 = \hat{L}_3$	✓ statement & reason ✓ statement & reason ✓ logical argument with reason ✓ statement & reason Letters must correspond ✓ conclusion (5)
2.3	In ΔLNM, $\hat{N}_1 = 90°$ (given) $LM^2 = LN^2 + NM^2$ (Pythagoras) $12^2 = LN^2 + x^2$ $LN^2 = 12^2 - x^2$ $\therefore \text{LN}= \sqrt{144\text{-}x^2}$	✓ statement & reason ✓ statement & reason ✓ substitution ✓answer (5)

QUESTION 3



3.1	$\hat{B}_1 = \hat{C}_2$. (tan chord theorem) $\hat{C}_2 = \widehat{D}_2$. (∠s in the same seg)	✓ reason ✓ reason (2)
3.2	$\therefore \hat{B}_1 = \widehat{D}_2$.	✓ conclusion (2)
3.3	Opposite AC: $\hat{B}_2 = \widehat{D}_1$. Opposite CD: $\hat{B}_3 = \hat{A}_1$. Opposite DB: $\hat{A}_2 = \hat{C}_1$. (∠s in the same seg)	✓ angle pair ✓ angle pair ✓ angle pair ✓ reason (4)
3.4	$\hat{A}_3 = \widehat{D}_1 + \widehat{D}_2$.. (ext ∠ of cyclic quad)	✓ $\widehat{D}_1 + \widehat{D}_2$.. ✓ reason (2)
3.5	$A\hat{C}D + A\hat{B}D = 180°$ $C\hat{A}B + B\widehat{D}C = 180°$ (opp ∠s of cyclic quad)	✓ statement ✓ statement ✓ reason (3)

QUESTION 4



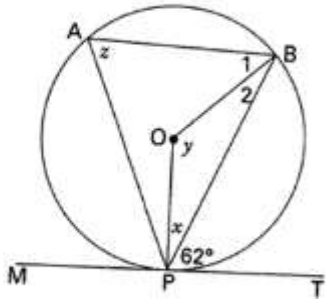
4.1	$P\hat{K}L = 90°$ (∠s in semi circle)	✓ statement ✓ reason (2)
4.2	$L\hat{N}P = 90°$ (∠s in semi circle) (opp ∠s of cyclic quad)	✓ statement ✓ reason ✓ reason (3)
4.3	$\hat{K}_2 + \hat{L}_1 + \hat{O}_1 = 180°$ (Int ∠s Δ) $\hat{O}_1 = 68°$ (given) $\hat{K}_2 = \hat{L}_1$ (∠s opp equal sides) $\therefore 2\hat{K}_2 + 68° = 180°$ or $2\hat{L}_1 + 68° = 180°$ $\therefore 2\hat{K}_2 = 112°$ or $2\hat{L}_1 = 112°$ $\therefore \hat{K}_2 = \hat{L}_1 = 56°$	✓ statement & reason ✓ statement & reason ✓ statement & reason ✓ substitution ✓ logical argument ✓ angle size (6)
4.4	$\hat{O}_1 = 2\hat{P}_1$. (∠ at centre = 2 × ∠ at circumference) $\hat{O}_1 = 68°$ (given) $\therefore 2\hat{P}_1. = 68°$ $\therefore \hat{P}_1 = 34°$.	✓ statement✓ reason ✓ statement & reason ✓ logical argument ✓ angle size (5)

SESSION 2

TOPIC: EUCLIDEAN GEOMETRY EXAM TYPE QUESTIONS 1: (GRADE 11)

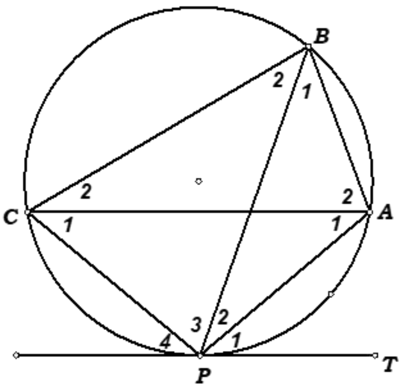
SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

QUESTION 1



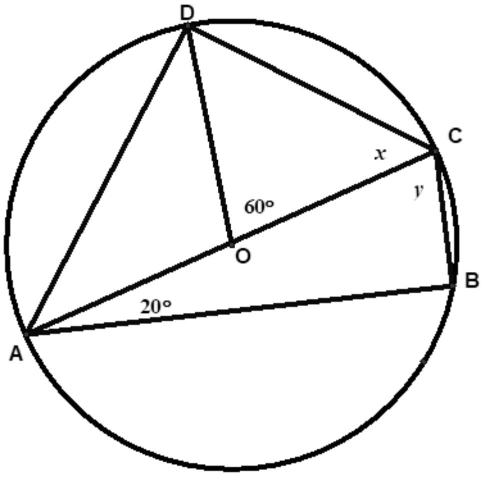
1	$x + 62° = 90°$ (radius ⊥ tangent) $\therefore x = 28°$ $x = \hat{B}_2 = 28°$ (∠s opp equal sides in Δ POB) $28° + y + \hat{B}_2 = 180°$ (Int ∠s of Δ POB) $\therefore y = 180° - 2 \times 28° = 124°$ $y = 2 \times z$ (∠ at centre = 2 × ∠ at circumference) $\therefore z = \frac{124°}{2} = 62°$	✓reason ✓angle ✓angle ✓ reason ✓reason ✓angle ✓reason ✓angle (8)

QUESTION 2



2.1	$\hat{P}_1 = \hat{B}_1$ (tan chord theorem) $\hat{B}_1 = \hat{C}_1$ (∠s in the same seg) $\therefore \hat{P}_1 = \hat{C}_1$ PC = PA (given) $\hat{C}_1 = \hat{A}_1$ (∠s opp equal sides) $\therefore \hat{P}_1 = \hat{A}_1$ $\hat{P}_4 = \hat{A}_1$ (tan chord theorem) $\therefore \hat{P}_1 = \hat{P}_4$ $\hat{P}_4 = \hat{B}_2$ (tan chord theorem) $\therefore \hat{P}_1 = \hat{B}_2$	✓ angle✓ reason ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle (10)
2.2	If $C\hat{P}A = 90°$ then $\hat{P}_1 = \hat{P}_4 = 45°$ (∠s on a str line) In order for $\hat{P}A = 90°$, CA must be a diameter of the circle.	✓∠s on a str line ✓ CA must be a diameter (2)

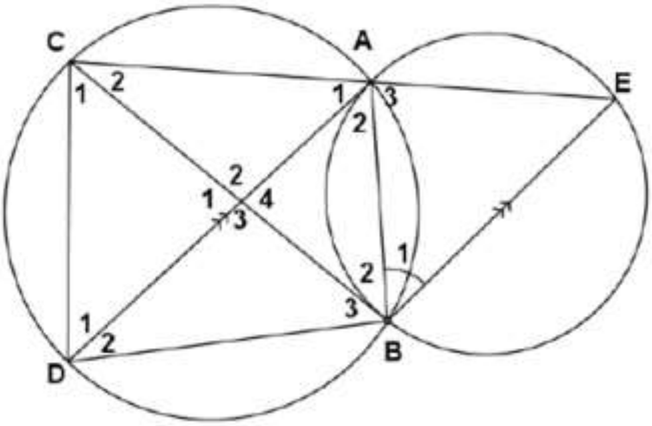
QUESTION 3



3.1	$\hat{B} = 90^0$ (∠s in semi circle) $\therefore y + 20° = 90°$ (Int ∠s Δ) $\therefore y = 90^o - 20^0$ $= 70^0$ OD = OC (radii) $\therefore x = O\hat{D}C$ (∠s opp equal sides) $x = \frac{180°-60°}{2} = 60^0$ (Int ∠s Δ)	✓statement ✓ reason ✓angle & reason ✓statement ✓ reason ✓angle & reason (6)
3.2	$O\hat{D}C = x = 60°$ (∠s opp equal sides) $\triangle ODC$ is an equilateral triangle (all angles equal 60°)	✓ equilateral ✓ reason (2)

SECTION B:	SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS 1

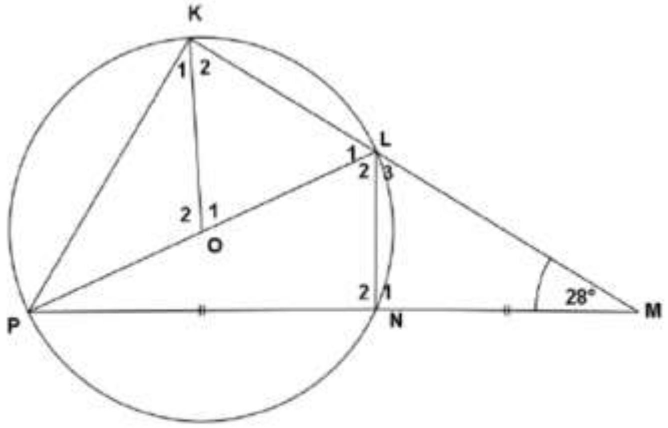
QUESTION 1 $A\hat{B}E = 40°$



$A\hat{B}E = 40°$

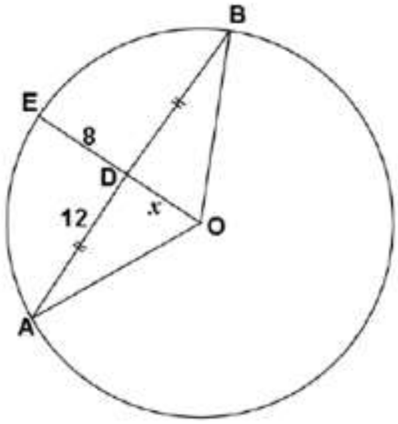
1.1	$A\hat{B}E = \hat{C}_2$ (tan chord theorem) $\hat{C}_2 = \hat{D}_2$ (∠s in the same seg) $\therefore A\hat{B}E = \hat{D}_2$ OR $A\hat{B}E = \hat{D}_2$ (tan chord theorem) $A\hat{B}E = \hat{A}_2$ (alt ∠s; AD||BE) $\hat{A}_2 = \hat{A}_1$ (AD bisects $C\hat{A}B$, given) $\therefore A\hat{B}E = \hat{A}_1$ $\hat{B}_3 = \hat{A}_1$ (∠s in the same seg) $\therefore A\hat{B}E = \hat{B}_3$ $\hat{C}_1 = \hat{A}_2$ (∠s in the same seg) $A\hat{B}E = \hat{A}_2$ (proved) $\therefore A\hat{B}E = \hat{C}_1$ $\hat{E} = \hat{A}_1$ (corresp ∠s; AD||BE) $A\hat{B}E = \hat{A}_1$ (proved) $\therefore A\hat{B}E = \hat{E}$	✓ $\hat{C}_2$ ✓reason ✓ $\hat{D}_2$ ✓reason ✓ $\hat{A}_2$ ✓reason ✓ $\hat{A}_1$ ✓reason ✓ $\hat{B}_3$✓reason ✓ $\hat{C}_1$✓reason ✓ $\hat{E}$✓reason (14)
1.2	$\hat{A}_3 + \hat{E} + A\hat{B}E = 180°$ (Int ∠s Δ) $\hat{A}_3 = 180° - 40° - 40°$ $\therefore \hat{A}_3 = 100°$	✓statement ✓reason ✓answer (2)

QUESTION 2



2.1	$L\hat{P}N$ lies in triangle LPN that has the diameter as a side, so you can find angle N_2. Another side of triangle LPN is PN which equals NM and LN the 3rd side is also a side of triangle NML. So use congruency. In ΔLPN and ΔLMN PN = NM (given) LN = LN (common) $\hat{N}_2 = 90°$ (∠s in semicircle) $\hat{N}_1 + \hat{N}_2 = 180°$ (∠s on a str line) $\therefore \hat{N}_1 = \hat{N}_2$ ∴ ΔLPN ≡ ΔLMN (SAS) $\therefore L\hat{P}N = \hat{M}$ $P\hat{M}K = 28°$ (given) $\therefore L\hat{P}N = 28°$	✓ given ✓ common ✓ ∠s in semicircle ✓ ∠s on a str line) ✓ SAS ✓ answer (6)
2.2	$\hat{L}_1 = \hat{P} + \hat{M}$ (ext ∠ of ΔLPM) $\hat{P} = \hat{M} = 28°$ (proved) $\therefore \hat{L}_1 = 28° + 28° = 56°$ $K\hat{O}P = 2 \times \hat{L}_1$ (∠ at centre = 2 × ∠ at circumference) $\therefore K\hat{O}P = 2 \times 56° = 112°$	✓ statement ✓reason ✓ statement ✓reason ✓ answer (5)

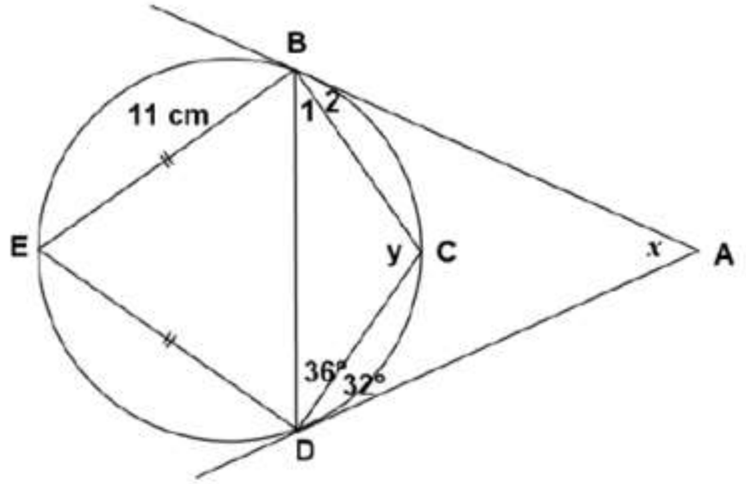
QUESTION 3



3.1	OB = OA = OE (radii) OE = $8 + x$ $\therefore$ OB = $8 + x$	✓ answer (1)
3.2	$O\hat{D}B = 90°$ (line from centre to midpt of chord) $OB^2 = OD^2 + DB^2$ (Pythagoras) $(8 + x)^2 = x^2 + 12^2$ $64 + 16x + x^2 = x^2 + 144$ $16x = 80$ $\therefore x = 5$ cm OB = $8 + x$ $\therefore$ OB = $8 + 5 = 13$ cm	✓ $90°$ ✓ formula ✓ substitution ✓ answer (4)

QUESTION 4

[Ignore EB = ED; and EB = 11cm on the diagram]



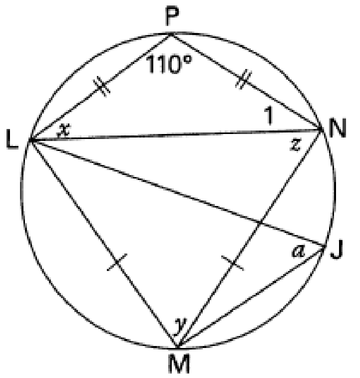
4.1	x lies in ΔBDA $B\widehat{D}A = 36° + 32° = 68°$ AB = AD (Tans from same pt) $B\widehat{D}A + D\widehat{B}A + x = 180°$(Int ∠s Δ) $68° + 68° + x = 180°$ $\therefore x = 44°$	✓ statement✓ reason ✓ answer (3)
4.2	y lies in ΔBDC $B\widehat{D}C = 36°$ (given) $\hat{B}_1 = 32°$ (tan chord theorem) $y = 180° - 36° - 32°$ (Int ∠s Δ) $\therefore y = 112°$	✓ tan chord theorem ✓ (Int ∠s Δ) ✓ answer (3)

SESSION NO: 3

TOPIC: EUCLIDEAN GEOMETRY EXAM TYPE QUESTIONS 2: (GRADE 11)

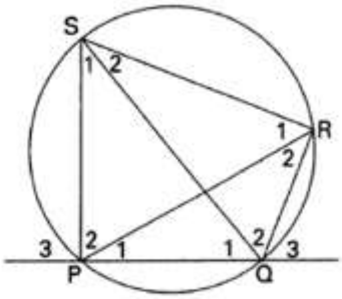
SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

QUESTION 1



1	$x = \widehat{N}_1$ (∠s opp equal sides in Δ PLN) $x + \widehat{N}_1 + 110° = 180°$ (Int ∠s Δ PLN) $\therefore x = \frac{(180° - 110°)}{2} = 35°$ $y + 110° = 180°$ (opp ∠s of cyclic quad) $\therefore y = 70°$ $M\hat{L}N = z$ (∠s opp equal sides in Δ LMN) $70° + M\hat{L}N + z = 180°$ (Int ∠s Δ LMN) $\therefore z = \frac{180° - 70°}{2} = 55°$ $a = 55°$ (∠s in the same segment)	✓statement ✓ reason ✓angle & reason ✓statement ✓ reason ✓statement ✓ reason ✓angle & reason ✓angle & reason (9)

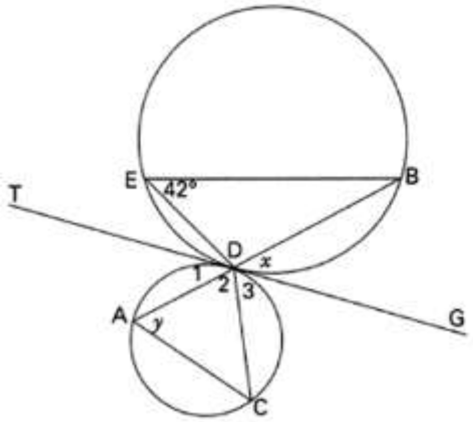
QUESTION 2



$\hat{P}_3 = 80°$; $\hat{Q}_3 = 70°$ and $\hat{S}_2 = 40°$

2.1	$\hat{Q}_3 = \hat{S}_1 + \hat{S}_2$ (ext ∠ of cyclic quad) $70° = \hat{S}_1 + 40°$ $\therefore \hat{S}_1 = 30°$	✓ reason ✓ answer (2)
2.2	$\hat{R}_2 = 30°$ (∠s in the same seg) $\hat{P}_3 = \hat{R}_1 + \hat{R}_2$ (ext ∠ of cyclic quad) $80° = \hat{R}_1 + 30°$ $\therefore \hat{R}_1 = 50°$	✓ answer ✓ reason ✓ reason ✓ answer (4)
2.3	$\hat{P}_1 = 40°$ (∠s in the same seg) $80° + \hat{P}_2 + 40° = 180°$ (∠s on a str line) $\therefore \hat{P}_2 = 60°$ **OR** $50° + 30° + \hat{P}_2 + 40° = 180°$ (opp ∠s of cyclic quad) $\therefore \hat{P}_2 = 60°$	✓ answer ✓ reason ✓ reason ✓ answer (4)

QUESTION 3

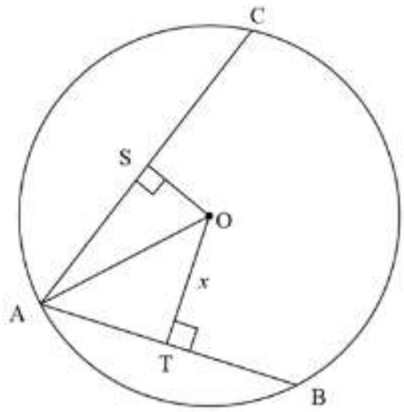


$\widehat{D}_2 = 70°$ and $\hat{E} = 42°$.

3.1	$x = 42°$ (tan chord theorem) $\widehat{D}_1 = 42°$ (vertically opposite angles) $\widehat{D}_3 + 42° + 70° = 180°$ ($\angle$s on a straight line) $\therefore \widehat{D}_3 = 68°$ $\therefore y = 68°$ (tan chord theorem)	✓ answer ✓ reason (2)
3.2	$x = \widehat{D}_1 = 42°$ (vert opp $\angle$s =) $\widehat{D}_3 + 42° + 70° = 180°$ ($\angle$s on a straight line) $\therefore \widehat{D}_3 = 68°$ $\therefore \widehat{D}_3 = y = 68°$ (tan chord theorem)	✓ answer ✓ reason ✓ reason ✓ answer ✓ answer ✓ reason (6)

SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS 2

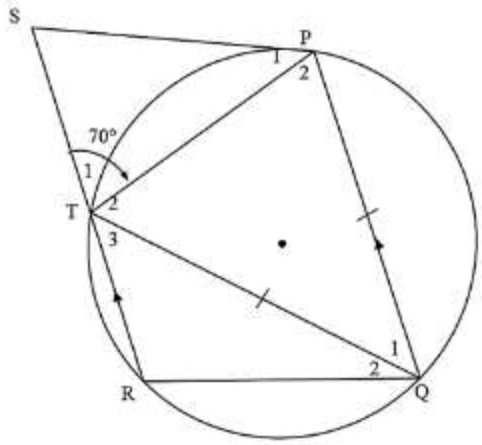
QUESTION 1



AB = 40 and AC = 48

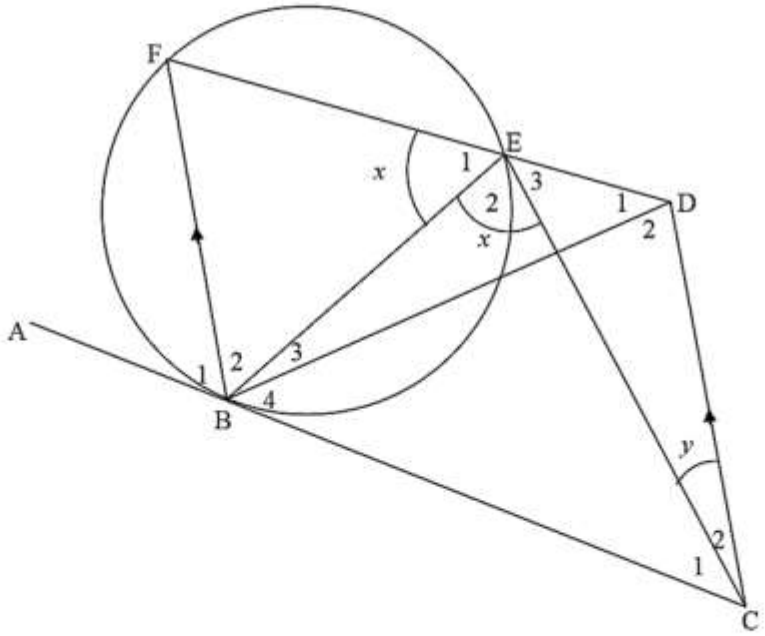
1.1	AT = 20 (line from centre ⊥ to chord)	✓ answer & reason (1)
1.2	Δ AOT is right-angled at T $\mathrm{AO}^2 = \mathrm{OT}^2 + \mathrm{AT}^2$ (Pythagoras) $25^2 = x^2 + 20^2$ $x = \sqrt{25^2 - 20^2} = 15$	✓ substitution & reason ✓ answer (2)
1.3	AS = 24 (line from centre ⊥ to chord) Δ AOS is right-angled at S $\mathrm{AO}^2 = \mathrm{OS}^2 + \mathrm{AS}^2$ (Pythagoras) $25^2 = \mathrm{OS}^2 + 24^2$ $\mathrm{OS} = \sqrt{25^2 - 24^2} = 7$ $\therefore \frac{\mathrm{OS}}{\mathrm{OT}} = \frac{7}{15}$.	✓ substitution & reason ✓ answer ✓ answer (3)

QUESTION 2



2.1	$\widehat{P}_2 = 70°$ Alternate ∠s PQ∥SR	✓ reason (1)
2.2(a)	$\hat{T}_2 = 70°$ (∠s opp equal sides) $\therefore \hat{Q}_1 = 180° - 2(70°)$ (Int ∠s Δ $= 40°$	✓ answer ✓ reason ✓ answer (3)
2.2(b)	$\widehat{P}_1 = 40$	✓ answer ✓ reason (2)

QUESTION 3



$\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$

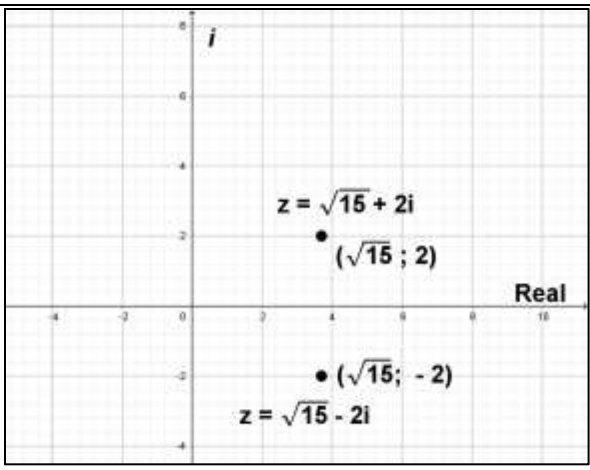
3.1 a)	$\hat{B}_1 = x$ (tan chord theorem)	✓ reason (1)
3.1 b)	$B\hat{C}D = \hat{B}_1$ (corresp ∠s FB∥DC)	✓ reason (1)
3.2	$\hat{D}_2 = \hat{E}_2$ (∠s in the same seg) *Note these equal x* $\hat{D}_2 = F\hat{B}D$ (alt ∠s BF∥CD) *So* $\hat{B}_3 + \hat{B}_2 = x$ *also*	✓ statement & reason ✓ statement & reason (2)
3.3	$B\hat{C}D = x$ from 3.1 b) $\therefore \hat{C}_1 + \hat{C}_2 = x$ $\hat{C}_2 = y$ (given) $\therefore \hat{C}_1 = x - y$ $\hat{B}_3 = \hat{C}_2 = y$ (∠s in the same seg) $\hat{B}_3 + \hat{B}_2 = x$ from 3.2 $\therefore y + \hat{B}_2 = x$ So $\hat{B}_2 = x - y = \hat{C}_1$	✓ statement ✓ statement & reason ✓ statement (3)

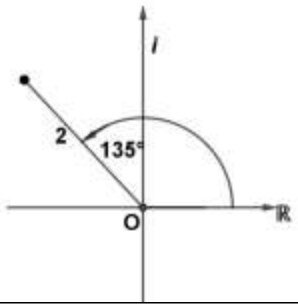
SESSION 4

TOPIC: COMPLEX NUMBERS OVERVIEW

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1.1 a)	5	✓	(1)
1.1 b)	$4i$	✓	(1)
1.2 a)	2	✓	(1)
1.2 b)	0	✓	(1)
2.1	$-2i.3i = -6i^2 = -6(-1) = 6$	✓$6(-1)$ ✓6	(2)
2.2	$-4i^2 \times i^3$ $= -4(-1)(-1)i$ $= -4i$	✓$-4(-1)(-1)i$ ✓$-4i$	(2)
2.3	$\left(1+\sqrt{5}i\right)^2$ $= \left(1+\sqrt{5}i\right)\left(1+\sqrt{5}i\right)$ $= 1+\sqrt{5}i+\sqrt{5}i+5i^2$ $= 1+2\sqrt{5}i+5(-1)$ $= -4+2\sqrt{5}i$	✓removing brackets ✓answer	(2)
2.4	$\left(1+\sqrt{5}i\right)\left(1-\sqrt{5}i\right)$ $= 1-5i^2$ $= 1-5(-1)$ $= 6$	✓removing brackets ✓answer	(2)
2.5	$\sqrt{-25}-\sqrt{-7}$ $= \sqrt{25}\times\sqrt{-1}\times\sqrt{7}\times\sqrt{-1}$ $= 5\sqrt{7} \times \left(\sqrt{-1}\right)^2$ $= 5\sqrt{7} \times -1$ $= -5\sqrt{7}$	✓splitting up ✓answer	(2)

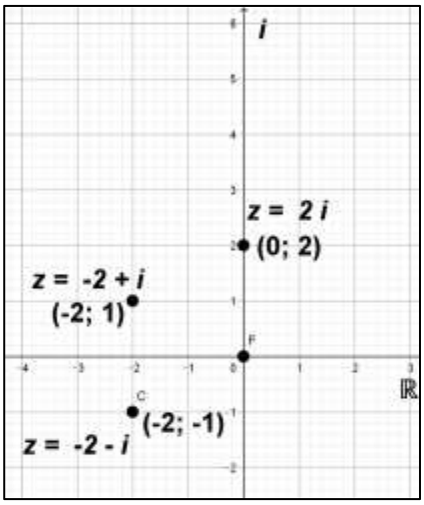
3	$\frac{3+2i}{-3i}$ $=\frac{3+2i}{-3i}\times\frac{-3i}{-3i}$ $=\frac{-9i-6i^2}{9i^2}$ $=\frac{-9i-6(-1)}{9(-1)}$ $=\frac{-9i}{-9}+\frac{6}{-9}$ $=i-\frac{6}{9}$ $=\frac{-2}{3}+i$ or $-0{,}67+i$	✓$\times\frac{-3i}{-3i}$ ✓ simplifying ✓answer (3)
4	$3x+14yi-6=6+7i$ $3x+14yi=12+7i$ *add 6 to both sides* $3x=12$ and $14yi=7i$ $\therefore x=4$ and $y=\frac{1}{2}$	✓$a+bi$ on both sides ✓ $x=4$ ✓$y=\frac{1}{2}$ (3)
5.1	$\bar{z}=\sqrt{15}+2i$	✓$\sqrt{15}+2i$ (1)
5.2		$z=\sqrt{15}-2i$ ✓point $\bar{z}=\sqrt{15}+2i$ ✓point ✓ labels on points and axes (3)

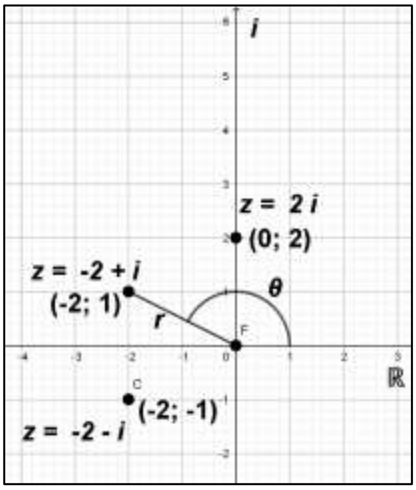
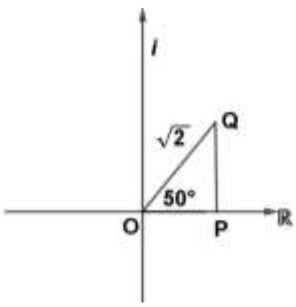
6.1	$z = 1 + 2\sqrt{2}\,i \quad a = 1 \qquad b = 2\sqrt{2}$ $r = \sqrt{a^2 + b^2}$ $= \sqrt{(1)^2 + (2\sqrt{2})^2}$ $= \sqrt{1+8}$ $= 3$ *check using your calculator*	✓ formula & substitution ✓answer (2)
6.2	$z = 1 + 2\sqrt{2}\,i$ lies in quadrant 1 *(a and b are positive),* $\theta = tan^{-1}\left(\frac{b}{a}\right)$ $\theta = tan^{-1}\left(\frac{2\sqrt{2}}{1}\right) = 70{,}53°.$ $\therefore \theta = 70{,}53°$ *check using your calculator*	✓ formula & substitution ✓ answer (2)
6.3	$z = 3(\cos 70{,}53° + i \sin 70{,}53°)$ Or $z = 3\ cis\ 70{,}53°$ Or $z = 3\lfloor\underline{70{,}53°}$	✓ format of answer ✓ 3 ✓ $70{,}53°$ (3)
7.1	$r = 2$ $\theta = 135°$ 	✓ $r = 2$ ✓ $\theta = 135°$ ✓ diagram (3)
7.2	$z = 2\ cis\ 135°$ $z = 2(\cos 135° + i \sin 135°)$ $z = 2\cos 135° + 2\sin 135° i$ $z = -\sqrt{2} + \sqrt{2}\,i$	✓expanding ✓$-\sqrt{2}$ ✓$+\sqrt{2}\,i$ (3)

SECTION B: SOLUTIONS TO PRACTICING THE BASICS OF COMPLEX NUMBERS

1.1 a)	3	✓	(1)
1.1 b)	$2i$	✓	(1)
1.2 a)	0	✓	(1)
1.2 b)	$-3i$	✓	(1)
2.1	$2i.3i = 6i^2 = 6(-1) = -6$	✓$6(-1)$ ✓ -6	(2)
2.2	$2(5i^3) = 10i^2.i = 10(-1)i = -10i$	✓$10(-1)i$ ✓ $-10i$	(2)
2.3	$2i^4 = 2(i^2)^2 = 2(-1)^2 = 2$	✓$2(-1)^2$ ✓2	(2)
3.1	$\sqrt{-144} = \sqrt{144} \times \sqrt{-1} = 12i$	✓working ✓answer	(2)
3.2	$\sqrt{-21} = \sqrt{21} \times \sqrt{-1} = \sqrt{21}\, i$	✓working ✓answer	(2)
3.3	$\sqrt{-75} = \sqrt{25} \times \sqrt{3} \times \sqrt{-1} = 5\sqrt{3}\, i$	✓working ✓answer	(2)
4.1	$(2+i)(3-i) = 2(3-i) + i(3-i)$ $= 6 - 2i + 3i - i^2$✓ $= 6 - 2i + 3i - (-1)$ $= 6 - 2i + 3i + 1$✓ $= 7 + i$✓	✓working ✓working ✓answer	(3)
4.2	$-\sqrt{144} - \sqrt{-1} + \sqrt{-25}$ $= -12 - i + 5i$ $= -12 + 4i$	✓-12 ✓$+5i$ ✓answer	(3)
4.3	$\frac{3 + 2i}{-3} = \frac{3}{-3} + \frac{2i}{-3}$ $= -1 - \frac{2}{3}i$	✓-1 ✓$+\frac{2i}{3}$ or $+\frac{2}{3}i$	(2)
5.1	$\bar{z} = 1 - 3i$	✓ $1 - 3i$	(1)

5.2	$\dfrac{3+i}{1+3i}$ $= \frac{3+i}{1+3i} \times \frac{1-3i}{1-3i}$ $\times$ *by complex conjugate* $= \frac{3(1-3i)+i(1-3i)}{1(1-3i)+3i(1-3i)}$ $= \frac{3-9i+i-3i^2}{1-3i+3i-9i^2}$ $= \frac{3-8i-3(-1)}{1-9(-1)}$ $= \frac{6-8i}{10}$ $\div$ *each term by 10* $= \frac{3}{5} - \frac{4}{5}i$ **or** $0{,}6 - 0{,}8\,i$	✓$\times \frac{1-3i}{1-3i}$ ✓$3 - 9i + i - 3i^2$ ✓$1 - 3i + 3i - 9i^2$ ✓$\frac{3}{5}$ or $0{,}6$ ✓ $-\frac{4}{5}i$ **or** $0{,}6 - 0{,}8\,i$ (5)
5.3	$\bar{z} \times \bar{z}$ $= (1-3i)(1-3i)$ $= 1(1-3i) - 3i(1-3i)$ $= 1 - 3i - 3i + 9i^2$ $= 1 - 3i - 3i + 9(-1)$ $= 1 - 3i - 3i - 9$ $= -8 - 6i$	✓substitution ✓simplification ✓answer (3)
6	$i - (x+iy) + 2(x+iy) - 3i = -2 + 7i$ $i - x - iy + 2x + 2yi - 3i = -2 + 7i$ $-x - iy + 2x + 2yi = -2 + 7i - i + 3i$ $-x + 2x + 2yi - iy = -2 + 7i - i + 3i$ $x + yi = -2 + 9i$ $x = -2$ and $yi = 9i \therefore y = 9$	✓simplification ✓$a + bi$ on both sides ✓ $x = -2$ ✓$y = 9$ (4)

7.1 to 7.3		$z = -2 + i$ ✓point ✓either label $z = 2i$ ✓point ✓either label $z = -2 - i$ ✓point ✓either label ✓axes labels (7)
8.1	$z_1 = -2 + i \quad a = -2 \qquad b = 1$ $r = \sqrt{a^2 + b^2}$ $= \sqrt{(-2)^2 + (1)^2}$ $= \sqrt{5}$ $= 2{,}24$ To check using your calculator see: **Converting from rectangular co-ordinates to polar co-ordinates:** to find the answer..	✓ formula & substitution ✓answer (2)
8.2	$z_1 = -2 + i$ lies in quadrant 3 so, $\theta = tan^{-1}\left(\frac{b}{a}\right)$ $z_1 = -2 + i$ lies in quadrant 2 so, $\theta = 180° - tan^{-1}\left(\frac{1}{+2}\right)$....*substitute positive values* $\therefore \theta = 153{,}43°$ If not told to find θ you could or write the answer as *arg(z)* $= 153{,}43°$ To check using your calculator see: **Converting from rectangular co-ordinates to polar co-ordinates:** to find the answer..	✓substitution ✓ answer (2)

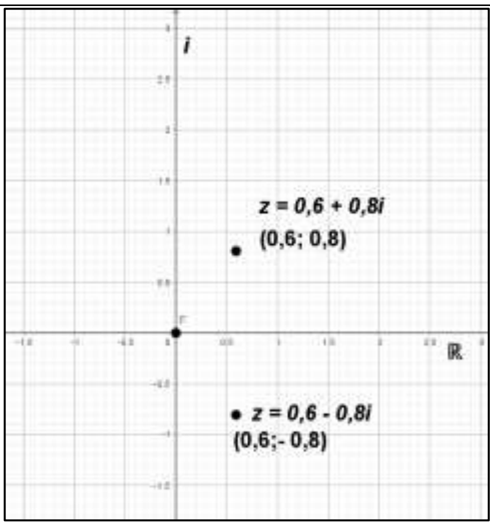
<table>
<tr><td>8.3</td><td>
</td><td>✓modulus
✓argument
Must be labelled (2)</td></tr>
<tr><td>9.1</td><td>$z = 5\ cis\ 240°$
$z = 5(\cos 240° + i \sin 240°)$
$z = 5\cos 240° + 5\sin 240° i$
first calculate $5\cos 240°$ and then calculate $5\sin 240°$
$z\ = \dfrac{-5}{2} + \dfrac{-\sqrt{5}}{2}\ i$
$\quad = -\frac{5}{2} - \frac{\sqrt{5}}{2}\ i$</td><td>✓expanding
✓$-\frac{5}{2}$
✓$-\frac{\sqrt{5}}{2}\ i$ (3)</td></tr>
<tr><td>9.2</td><td>$z = \sqrt{2}|\underline{50°}$


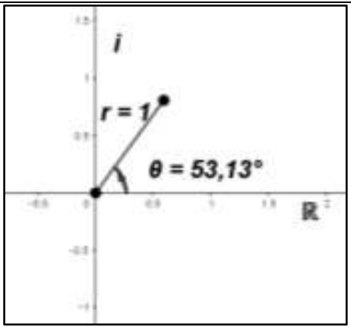
$\cos 50° = \frac{OP}{\sqrt{2}}$
$\therefore OP = \sqrt{2}\cos 50° = 0{,}91$
And $\sin 50° = \frac{QP}{\sqrt{2}}$
$\therefore QP = \sqrt{2}\sin 50° = 1{,}08$
The rectangular (or Cartesian) form is
$z = OP + QP\ i$
$\therefore z = 0{,}91 + 1{,}08\ i$</td><td>✓diagram
✓$\cos 50° = \frac{OP}{\sqrt{2}}$
✓$\sin 50° = \frac{QP}{\sqrt{2}}$
✓ answer (4)</td></tr>
</table>

SESSION 5

TOPIC: EXAM TYPE QUESTIONS ON COMPLEX NUMBERS

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1.1	$\sqrt{-5}-\sqrt{-2}+\sqrt{-4}$ $=\sqrt{5}\sqrt{-1}-\sqrt{2}\sqrt{-1}+\sqrt{4}\sqrt{-1}$ $=\sqrt{5}i-\sqrt{2}i+2i$	✓splitting up ✓ answer (2)
1.2	$(2i)^3+14i^2-(9+2i)$ $=8i^3+14i^2-9-2i$ $=8(-1)i+14(-1)-9-2i$ $=-8i-14-9-2i$ $=-10i-23$	✓ remove brackets ✓$-8i-14-9-2i$ ✓ answer (3)
1.3	$(1+i)(5-3i)$ $=(1)(5-3i)+i(5-3i)$ $=5-3i+5i-3i^2$ $=5-3i+5i-3(-1)$ $=2i+8$	✓ remove brackets ✓ answer (2)
2.1	$\bar{z}=\frac{3}{5}-\frac{4}{5}i$	✓ $\frac{3}{5}$ ✓ $-\frac{4}{5}i$ (2)
2.2		$z=\frac{3}{5}+\frac{4}{5}i=0{,}6+0{,}8i$ ✓point & either label $\bar{z}=\frac{3}{5}-\frac{4}{5}i=0{,}6-0{,}8i$ ✓point &either label ✓axes labels (3)

2.3	$a = \frac{3}{5}$ and $b = \frac{4}{5}$ $r = \sqrt{a^2 + b^2}$ $= \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$ *or substitute 0,6 and 0,8* $= 1$ **Or** by using the calculator method	✓ formula & substitution ✓answer (2) OR ✓✓ answer if calculator method used
2.4	$z = \frac{3}{5} + \frac{4}{5}i$ lies in quadrant 1 so, $\theta = tan^{-1}\left(\frac{0,8}{0,6}\right)$.... *or substitute* $\frac{4}{5}$*and* $\frac{3}{5}$ $\therefore \theta = 53,13°$ **Or** by using the calculator method	✓substitution ✓ answer (2) OR ✓✓ answer if calculator method used
2.5		✓ r and label ✓ $\theta°$ and label Length and angle size should be correct (2)
2.6	$z = 1\ (\cos 53,13° + i \sin 53,13°)$ or $z = 1\ \text{cis}\, 53,13°$ or $1\underline{\vert 53,13°}$	✓ format ✓ substitution (2)

3	$\dfrac{3-2i}{i-3}$ $=\dfrac{3-2i}{i-3}\times\dfrac{i+3}{i+3}$ $=\frac{(3-2i)(3+i)}{(i-3)(i+3)}$ *swop $i+3$ to $3+i$ so both in same order* $=\dfrac{3(3+i)-2i(3+i)}{i(i+3)-3(i+3)}$ $=\dfrac{9+3i-6i-2(-1)}{(-1)+3i-3i-9}$ $=\dfrac{11-3i}{-10}$ $=\frac{-11}{10}+\frac{3i}{10}$ or $-1{,}1+0{,}3i$	✓$\times\frac{i+3}{i+3}$ ✓simplify ✓$\frac{-11}{10}$ or $-1{,}1$ ✓$+\frac{3i}{10}$ or $+0{,}3i$ (4)
4	$5(\cos 0° + i\sin 0°)$ $=5\times 1+5\times 0i$ $=5$	✓ format ✓ answer (2)
5	$(1-2i)(x+3i)=y-3i$ $1(x+3i)-2i(x+3i)=y-3i$ $x+3i-2xi-6(-1)=y-3i$ $x+6-2xi=y-6i$ $x+6=y$ and $-2xi=y-6i$ $\therefore x=3$ substitute into $x+6=y$ $(3)+6=y$ $\therefore y=9$	✓simplification ✓$a+bi$ on both sides ✓ $x=3$ ✓$y=9$ (4)

6	$E = I.Z$ $= (2 + 6i)(3 - 4i)$ $= 6 - 8i + 18i - 24(-1)$ $= 30 + 10i$ volts	✓substitution ✓simplification ✓$30 + 10i$ ✓volts (4)

SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS

1.1	$-\sqrt{16}-\sqrt{-1}+\sqrt{-81}$ $=-4-\sqrt{-1}+\sqrt{81\times -1}$ $=-4-i+9i$ $=-4+8i$	✓splitting up ✓ -4 ✓ $+8i$ (3)
1.2	$3+14i^3-6-(6+7i)$ $=3+14i^3-6-6-7i$ $=3+14(-1)i-6-6-7i$ $=3-14i-12-7i$ $=-9-21i$	✓ splitting up & remove brackets ✓ -9 ✓ $-21i$ (3)
1.3	$\frac{7+i}{2}=\frac{7}{2}+\frac{i}{2}$ $=\frac{7}{2}+\frac{1}{2}i$ **or** $3{,}5\ +0{,}5\ i$	✓ split up ✓ answer (2)
2.1	$a=-2$ and $b=2$ $z=-2a+2i$	✓ $-2a$ ✓ $2i$ (2)
2.2	$a=-2$ and $b=2$ $r=\sqrt{a^2+b^2}$ $=\sqrt{(-2)^2+(2)^2}$ $=\sqrt{8}$ **or** $2{,}83$	✓ formula & substitution ✓answer (2) OR ✓✓ answer if calculator method used
2.3	$z=-2a+2i$ lies in quadrant 2 so, $\theta=180°-tan^{-1}\left(\frac{+2}{2}\right)$....*substitute positive values* $\therefore \theta=135°$ **Or** by using the calculator method	✓substitution ✓ answer (2) OR ✓✓ answer if calculator method used

2.4	$z = \sqrt{8}\ (\cos 135° + i \sin 135°)$ or $z = \sqrt{8}\ \text{cis}\, 135°$ or $\sqrt{8}\lvert\underline{135°}$ **OR** $z = 2{,}83(\cos 135° + i \sin 135°)$ or abbreviations	✓ format ✓ substitution (2)
3	$(1 + i)(5 + 3i)$ $= 1(5 + 3i) + i(5 + 3i)$ $= 5 + 3i + 5i + 3i^2$ $= 5 + +3i + 5i + 3(-1)$ $= 5 + +3i + 5i - 3$ $= 2 + 8i$	✓removing brackets ✓ *2* ✓$+8i$ (3)
4	$2\sqrt{2}(\cos 315° + i \sin 315°)$ $= 2\sqrt{2} \times \frac{\sqrt{2}}{2} + 2\sqrt{2} \times \left(\frac{-\sqrt{2}}{2}\right)$ $= 2 - 2i$	✓2 ✓$-2i$ (2)
5	$y - 4i = (2 - i)(3x + i)$ $y - 4i = 6x + 2i - 3xi - (-1)$ $y - 4i = 6x + 2i - 3xi + 1$ $y - 6i = 6x + 1 - 3xi$ So $y = 6x + 1$ and $6i = -3xi$ i.e. $6 = -3x$ $\therefore -2 = x$ substitute into $y = 6x + 1$ $y = 6(-2) + 1$ $\therefore y = 11$	✓simplification ✓$a + bi$ on both sides ✓ $x = -2$ ✓$y = 11$ (4)

6	$z_1 = 2 - 3i$ and $z_2 = 1 + 4i$ $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$ $= \frac{(2-3i)(1+4i)}{2-3i+1+4i}$ $= \frac{2+4i-3i-4(-1)}{3+i}$ $= \frac{2+4i-3i+4}{3+i}$ $= \frac{6+i}{3+i} \times \frac{3-i}{3-i}$ $= \frac{18-6i+3i-(-1)}{9-(-1)}$ $= \frac{19-3i}{10}$ **or** $1{,}9 - 0{,}3i$ ohms	✓substitution ✓$\frac{2+4i-3i+4}{3+i}$ ✓$\times \frac{3-i}{3-i}$ ✓answer& unit (4)

SESSION NO: 6

TOPIC: CIRCLES, ANGLES AND ANGULAR MOVEMENT

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1.1	45 rev/min = $\frac{45\ rev}{60\ s}$ = 0,75 rev/s	✓correct method (1)
1.2	***n*** = 0,75 rev/s $\omega = 2\pi n$ $= 2 \times \pi \times 0{,}75$ $= 1{,}5\pi$ ω = 4,71 rad/s	✓correct formula ✓ substitution ✓ answer & unit (3)
2.1	***s*** = ***vt*** = 2(10) ***s*** = 20 ***m***	✓correct formula & substitution ✓ answer & unit (2)
2.2	***D = 0,4m ; v = 2m*** ***v*** = πDn 2 = π(0,4)(***n***) ***n*** = $\frac{2}{0{,}4\pi}$ ***n*** = 1,59 ***r/s***	✓correct formula & substitution ✓ for making n the subject of the formula ✓ answer & unit (3)

2.3	1 rev = 2π radians 12 rev = 12 × 2π radians 12 rev = 75,398 radians $\omega = \frac{\theta}{t}$ t = $\frac{\theta}{\omega}$ …….(1) But: $\omega = 2\pi n$ = 2π(1,59) ω = 9,99rad/s …….(2) Substitute (2) into (1) ***t*** = $\frac{75{,}398}{9{,}99}$ ***t*** = 7,55 ***s***	✓12 × 2π radians or 75,398 radians ✓ for making t the subject of the formula ✓ ω = 9,99 ✓***t*** = 7,55 (4)
3.1	s = $r\theta$ 430 cm = ***r***(2,8) ***r*** = 153,57 cm	✓ answer (1)
3.2	Area of sector = $\frac{\theta r^2}{2}$ = $\frac{2{,}8\times(153{,}57)^2}{2}$ = 33017,86 cm^2	✓correct formula & substitution ✓ answer (2)
4.1	CA^2 = AO^2 – OC^2 …..Theorem of Pythagoras = $(10)^2 - (6)^2$ CA = 8 m	✓statement & reason ✓ answer (2)
4.2	AB = 16 = ***x*** ; D = 20 ; ***h*** = ? 20 = h + $\frac{16^2}{4h}$ 80***h*** = 4***h***^{2} + 256 4***h***^{2} – 80***h*** + 256 = 0 ***h***^{2} – 20***h*** + 64 = 0 (***h*** – 16)(***h*** – 4) = 0 ***h*** = 16 OR ***h*** = 4	✓correct substitution ✓for quadratic equation in standard form ✓factors or correct substitution into the quadratic formula ✓ ***h*** = 16 ✓ ***h*** = 4 (5)

SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS 2

1.1	$s = r\theta$ $50 = r \times (1{,}3)$ $\frac{50}{1{,}3} = r$ $\therefore r = 38{,}461....$ $= 38{,}46$ cm	✓ formula & substitution ✓ Answer (2)
1.2	Area of sector $= \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 38^2 \times 1{,}3$ $= 938{,}6 \text{ cm}^2$	✓ substitution ✓ answer & unit (2)
2	$\theta = 35°$ $d = 32\ cm$ $\therefore r = 16\ cm$ $s = r\theta$ $= 16 \times 35° \times \frac{\pi}{180°}$ $= \frac{28}{9}\pi$ $= 9{,}773...$ $\therefore s = 9{,}77\ cm$	✓ substitution into correct formula ✓ answer & unit (2)
3	$D = h + \frac{x^2}{4h}$ $25 = h + \frac{(20)^2}{4h}$ $100h = 4h^2 + 400$ $0 = 4h^2 - 100h + 400 \ldots\ldots.. \div 4$ $0 = h^2 - 25h + 100$ $h = \frac{-(-25)\pm\sqrt{(-25)^2-4(1)(100)}}{2(1)}$ or $(h-20)(h-5) = 0$ $h = 20 \text{ cm}$ or $h = 5 \text{ cm}$	✓ formula ✓ substitution ✓ standard form ✓ substitution into quadratic formula or factorising ✓ answers & unit (5)
4	Area of sector $= \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times 35^2 \times 75 \times \frac{\pi}{180} \text{ mm}^2$ $= 801{,}760\ 6251 \text{ mm}^2$ $\approx 802 \text{ mm}^2$	✓ substitution into correct formula ✓ $75 \times \frac{\pi}{180}$ ✓ answer & unit (3)

5.1	500 rev/ minute $= \frac{540}{60}$ $= 9$ rev/s	✓ answer (1)
5.2	$w = 2\pi n$ $= 2 \times \pi \times 9$ rev/s $= 18\pi$ rev/s $= 56{,}55$ rad/s	✓ correct formula ✓ substitution ✓ answer & unit (3)
6.1	For n revolutions per second $\boldsymbol{v} = \boldsymbol{\pi D n}$ $\mathrm{m.}s^{-1}$ For n revolutions per hour $\boldsymbol{v} = \boldsymbol{\pi D n}$ $km.h^{-1}$ Train does $\frac{2}{3}$ revolution in 40 minutes $\times 3$ Train does 2 revolutions in120minutes $\div 2$ Train does 1 revolution in 60 minutes We are also given that $D = 8\ km$ So, $v = \pi D n$ $= \pi \times 8$ km $\times 1$ rev/h $= 8\pi$ km/h $= 25{,}132...$ km/h $= 25{,}13$ km/h	✓ time for 1 revolution ✓ formula ✓ substitution ✓ answer & unit (4)
6.2	From 5.1 the train does 1 revolution in 60 minutes (1 hour),so it will take the train 4 hours to do 4 revolutions **OR** Using $v = \pi D n$ $8\pi = \pi \times 8 \times n$ *do not use rounded off answers* $n = \frac{8\pi}{\pi \times 8}$ $n = 1$ rev /hour $\therefore$ 4 hours for 4 revolutions **OR** $v = \frac{s}{t}$ so $t = \frac{s}{v} = \frac{4\pi \times 8}{8\pi} =$ 4 hours	✓ explanation ✓ 4 hours OR ✓ substitution into correct formula ✓ 4 hours (2)
7	4 km = 400 000 cm 1 852 revolutions = 400 000 cm $\therefore$ 1 revolution = $\frac{400\ 000}{1\ 852}$ cm $C = \pi d$ $\therefore d = \frac{C}{\pi}$ $= \frac{\frac{400\ 000}{1\ 852}}{\pi}$ $= 68{,}749 \ldots$ ≈ 69 cm	✓ $C = \pi d$ ✓ substitution ✓ answer & unit (3)