

SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2019



GRADE 12

SUBJECT: MATHEMATICS

LEARNER NOTES

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SESSION NO: 1

TOPIC: SEQUENCES AND SERIES

Learner Note: Sequences and series is an exciting part of the curriculum. Make sure you know the difference between arithmetic and geometric sequences. You also need to know the relevant formulae for finding specific terms, and the sum of a certain number of terms. The sum to infinity is an important concept as well as real world applications of the formulae.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

- (a) Consider the sequence: -2; 3; 8; 13; 18; 23; 28; 33; 38; ...
- (1) Determine the 100th term. (2)
 - (2) Determine the sum of the first 100 terms. (2)
- (b) The 13th and 7th terms of an arithmetic sequence are 15 and 51 respectively. Which term of the sequence is equal to -21 (6)

QUESTION 2

In a geometric sequence, the 6th term is 243 and the 3rd term is 72.

Determine:

- (a) The constant ratio. (4)
- (b) The sum of the first 10 terms. (4)

QUESTION 3 (DoE Nov 2008 Paper 1)

Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \dots$

- (a) If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)
- (b) Calculate the sum of the first 50 terms of the sequence. (7)

QUESTION 4

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \dots$

- (a) Determine the n^{th} term of the series. (3)
- (b) For what value(s) of x will the series converge? (2)
- (c) Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

QUESTION 5

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

- (a) Calculate the value of the 5th term, T_5 , of this sequence. (2)
- (b) Determine the n^{th} term, T_n , in terms of n . (2)
- (c) Explain why the infinite series $10 + 5 + 2,5 + 1,25 + \dots$ converges. (2)
- (d) Determine $S_{\infty} - S_n$ in the form ab^n , where S_n is the sum of the first n terms of the sequence. (4)

QUESTION 6

Consider the series: $S_n = -3 + 5 + 13 + 21 + \dots$ to n terms.

- (a) Determine the general term of the series in the form $T_k = bk + c$. (2)
- (b) Write S_n in sigma notation. (2)
- (c) Show that $S_n = 4n^2 - 7n$. (3)
- (d) Another sequence is defined as:

$$Q_1 = -6$$

$$Q_2 = -6 - 3$$

$$Q_3 = -6 - 3 + 5$$

$$Q_4 = -6 - 3 + 5 + 13$$

$$Q_5 = -6 - 3 + 5 + 13 + 21$$

(1) Write down a numerical expression for Q_6 . (2)

(2) Calculate the value of Q_{129} . (3)

QUESTION 7

Given $f(x) = 3(x - 1)^2 + 5$ and $g(x) = 3$

(a) Is it possible for $f(x) = g(x)$? Given a reason for your answer. (2)

(b) Determine the value(s) of k for which $f(x) = g(x) + k$ has two unequal real roots. (2)

QUESTION 8

Given arithmetic series $18 + 24 + 30 + \dots + 300$

(a) Determine the number of terms in the series. (3)

(b) Calculate the sum of this series. (2)

(c) Calculate the sum of all the whole numbers up to and including 300 that are not divisible by six. (4)

QUESTION 9

The sum of n terms is given by $S_n = \frac{n}{2}(1 + n)$, find T_5 . (3)

SECTION B: NOTES ON CONTENT

Arithmetic Sequences and Series

An arithmetic sequence or series is the linear number pattern discussed in Grade 10. We have a formula to help us determine any specific term of an arithmetic sequence. We also have formulae to determine the sum of a specific number of terms of an arithmetic series.

The formulae are as follows:

$$T_n = a + (n-1)d \quad \text{where } a = \text{first term and } d = \text{constant difference}$$

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{where } a = \text{first term and } d = \text{constant difference}$$

$$S_n = \frac{n}{2}[a + l] \quad \text{where } l \text{ is the last term}$$

Geometric Sequences and Series

We have a formula to help us determine any specific term of a geometric sequence. We also have formulae to determine the sum of a specific number of terms of a geometric series.

The formulae are as follows:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{where } r \neq 1$$

Convergent geometric series

Consider the following geometric series:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

We can work out the sum of progressive terms as follows:

$$S_1 = \frac{1}{2} = 0,5 \quad (\text{Start by adding in the first term})$$

$$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0,75 \quad (\text{Then add the first two terms})$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} = 0,875 \quad (\text{Then add the first three terms})$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16} = 0,9375 \quad (\text{Then add the first four terms})$$

If we continue adding progressive terms, it is clear that the decimal obtained is getting closer and closer to 1. The series is said to converge to 1. The number to which the series converges is called the sum to infinity of the series.

There is a useful formula to help us calculate the sum to infinity of a convergent geometric series.

The formula is $S_{\infty} = \frac{a}{1-r}$

If we consider the previous series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

It is clear that $a = \frac{1}{2}$ and $r = \frac{1}{2}$

$$S_{\infty} = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

A geometric series will converge only if the constant ratio is a number between negative one and positive one.

In other words, the sum to infinity for a given geometric series will exist only if $-1 < r < 1$.

If the constant ratio lies outside this interval, then the series will not converge.

SECTION C: ACTIVITIES

QUESTION 1

The 19th term of an arithmetic sequence is 11, while the 31st term is 5.

- Determine the first three terms of the sequence. (5)
- Which term of the sequence is equal to -29 ? (3)

QUESTION 2

Given: $\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + \dots + \frac{180}{181}$

- Calculate the sum of the given series. (4)
- Hence calculate the sum of the following series:

$$\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{181} + \frac{2}{181} + \dots + \frac{180}{181}\right) \quad (4)$$

QUESTION 3 (DoE Feb 2009 Paper 1)

The following is an arithmetic sequence: $1 - p ; 2p - 3 ; p + 5 ; \dots$

- (a) Calculate the value of p . (3)
- (b) Write down the value of:
 - (1) The first term of the sequence (1)
 - (2) The common difference (2)
- (c) Explain why none of the numbers in this arithmetic sequence are perfect squares. (2)

QUESTION 4

In a geometric sequence in which all terms are positive, the sixth term is $\sqrt{3}$ and the eighth term is $\sqrt{27}$. Determine the first term and constant ratio. (7)

QUESTION 5

- (a) Determine n if: $\sum_{r=1}^n (6r - 1) = 456$ (7)
- (b) Prove that: $\sum_{k=3}^n (2k - 1)n = n^3 - 4n$. (6)
- (c) Write the following series in sigma notation: $2 + 5 + 8 + 11 + 14 + 17$ (4)

QUESTION 6

Consider the series $\sum_{n=1}^{\infty} 2\left(\frac{1}{2}x\right)^n$

- (a) For which values of x will the series converge? (3)
- (b) If $x = \frac{1}{2}$, calculate the sum to infinity of this series. (3)

	$\therefore a - 72 = 15$ $\therefore a = 87$ $T_n = -21$ $\therefore a + (n-1)d = -21$ $\therefore 87 + (n-1)(-6) = -21$ $\therefore 87 - 6n + 6 = -21$ $\therefore n = 19$ $\therefore T_{19} = -21$	
2(a)	$T_6 = 243$ and $T_3 = 72$ $a.r^5 = 243 \dots A$ $a.r^2 = 72 \dots B$ $a.r^5 = 243 \dots A$ $a.r^2 = 72 \dots B$ $\therefore r^3 = \frac{27}{8} \dots A \div B$ $\therefore r = \frac{3}{2}$	$\checkmark a.r^5 = 243$ $\checkmark a.r^2 = 72$ $\checkmark r^3 = \frac{27}{8}$ $\checkmark r = \frac{3}{2}$ (4)
2(b)	$\therefore a \left(\frac{3}{2} \right)^5 = 243$ $\therefore a = 32$ $\therefore S_{10} = \frac{32 \left(\left(\frac{3}{2} \right)^{10} - 1 \right)}{\frac{3}{2} - 1} = 3626,5625$	$\checkmark a \left(\frac{3}{2} \right)^5 = 243$ $\checkmark a = 32$ $\checkmark S_{10} = \frac{32 \left(\left(\frac{3}{2} \right)^{10} - 1 \right)}{\frac{3}{2} - 1}$ $\checkmark$ answer (4)
3(a)	$\frac{1}{16} ; 13$	$\checkmark \checkmark$ answers (2)
3(b)	$S_{50} = 25 \text{ terms of } 1^{\text{st}} \text{ sequence} + 25 \text{ terms of } 2^{\text{nd}} \text{ sequence}$ $S_{50} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{to 25 terms} \right) + (4 + 7 + 10 + 13 + \dots \text{to 25 terms})$ $S_{50} = \frac{\frac{1}{2} \left(\left(\frac{1}{2} \right)^{25} - 1 \right)}{\frac{1}{2} - 1} + \frac{25}{2} [2(4) + 24(3)]$ $S_{50} = 0,999999\dots + 1000$ $S_{50} = 1001,00$	$\checkmark$ separating into an arithmetic and geometric series $\checkmark \checkmark \frac{\frac{1}{2} \left(\left(\frac{1}{2} \right)^{25} - 1 \right)}{\frac{1}{2} - 1}$ $\checkmark$ correct formulae $\checkmark \checkmark \frac{25}{2} [2(4) + 24(3)]$ $\checkmark$ answer (7)

4(a)	$T_n = (8x^3) \left(\frac{1}{2}x \right)^{n-1}$	✓ correct formula ✓ $a = 8x^3$ ✓ $r = \frac{1}{2}x$ (3)
4(b)	$-1 < \frac{x}{2} < 1$ $= -2 < x < 2$	✓ $-1 < \frac{x}{2} < 1$ ✓ $-2 < x < 2$ (2)
4(c)	$S_\infty = \frac{a}{1-r}$ $\therefore S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $\therefore S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$ $\therefore S_\infty = 72$	✓ correct formula ✓ substitution ✓ answer (3)
5(a)	$r = \frac{T_2}{T_1}$ $= \frac{5}{10}$ $= \frac{1}{2}$ $T_5 = 1,25 \left(\frac{1}{2} \right)$ $= \frac{5}{8} \text{ or } 0,625$	$\checkmark r = \frac{1}{2}$ $T_5 = 10 \left(\frac{1}{2} \right)^4$ $= \frac{5}{8} \text{ or } 0,625$ ✓ answer (2)
5(b)	$T_n = 10 \left(\frac{1}{2} \right)^{n-1}$	✓ substitutes $a = 10$ into GP formula $r = \frac{1}{2}$ ✓ substitutes into GP formula (2)
5(c)	$r = \frac{1}{2}$ $-1 < r < 1$ Therefore the sequence converges/Die ry konvergeer	✓ $-1 < r < 1$ ✓ show that $r = \frac{1}{2}$ is $-1 < r < 1$ (2)

5(d)

$$\begin{aligned}
 S_{\infty} - S_n &= \frac{a}{1-r} - \frac{a(1-r^n)}{1-r} \\
 &= \frac{10}{1-\frac{1}{2}} - \frac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}} \\
 &= 20 - 20\left(1-\frac{1}{2}^n\right) \\
 &= 20 - 20 + 20 \times \left(\frac{1}{2}\right)^n \\
 &= 20 \times \left(\frac{1}{2}\right)^n
 \end{aligned}$$

OR/OF

$$\begin{aligned}
 S_{\infty} - S_n &= T_{n+1} + T_{n+2} + T_{n+3} + \dots \\
 &= 10\left(\frac{1}{2}\right)^n \left[1 + \frac{1}{2} + \frac{1}{4} + \dots\right] \\
 &= 10\left(\frac{1}{2}\right)^n \left[\frac{1}{1-\frac{1}{2}}\right] \\
 &= 20 \times \left(\frac{1}{2}\right)^n
 \end{aligned}$$

OR/OF

$$\begin{aligned}
 &\frac{10}{1-\frac{1}{2}} \\
 \checkmark &\frac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}
 \end{aligned}$$

$$\checkmark -20 + 20 \times \left(\frac{1}{2}\right)^n$$

✓answer
(4)

✓constructing the series

$$\checkmark 10\left(\frac{1}{2}\right)^n \left[1 + \frac{1}{2} + \frac{1}{4} + \dots\right]$$

$$\checkmark \frac{1}{1-\frac{1}{2}}$$

✓answer
(4)

	$S_{\infty} - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{a - a + ar^n}{1-r}$ $= \frac{ar^n}{1-r}$ $= \frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ $= 20 \times \left(\frac{1}{2}\right)^n$	$\frac{a - a + ar^n}{1-r}$ $\frac{ar^n}{1-r}$ $10\left(\frac{1}{2}\right)^n$ $\frac{1}{2}$ ✓ answer (4)
6(a)	$T_k = a + (k-1)d$ $= -3 + (k-1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓ answer (2)
6(b)	$\sum_{k=1}^n (8k-11) \quad \text{OR/OF} \quad \sum_{k=0}^{n-1} (8(k+1)-11) = \sum_{k=0}^{n-1} (8k-3)$	✓ for general term ✓ lower and upper values in sigma notation (2)
6(c)	$S_n = \frac{n}{2} [2a + (n-1)d]$ $= \frac{n}{2} [2(-3) + (n-1)(8)]$ $= \frac{n}{2} [-6 + 8n - 8]$ $= \frac{n}{2} [8n - 14]$ $= n(4n - 7)$ $= 4n^2 - 7n$ OR/OF	✓ formula ✓ substitution $\frac{n}{2} [8n - 14]$ ✓ (3)

	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ OR/OF $S_n = \frac{n}{2}[a + l]$ $= \frac{n}{2}[-3 + 8n - 11]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ OR/OF $\begin{array}{ccccccc} S_1 & & S_2 & & S_3 & & S_4 \\ -3 & & 2 & & 15 & & 36 \\ & \diagdown & & \diagup & & \diagdown & \\ & 5 & & 13 & & 21 & \\ & \diagup & & \diagdown & & \diagup & \\ & 8 & & 8 & & & \end{array}$ $S_n = an^2 + bn + c$ $a = 4$ $S_1 = 4 + b + c = -3 \quad b + c = -7 \dots\dots\dots(1)$ $S_2 = 16 + 2b + c = 2 \quad 2b + c = -14 \dots\dots\dots(2)$ $b = -7 \dots\dots\dots(2) - (1)$ $c = 0$ Hence $S_n = 4n^2 - 7n$	✓ formula ✓ substitution $\frac{n}{2}[8n - 14]$ (3) ✓ formula ✓ substitution $\frac{n}{2}[8n - 14]$ (3) ✓ calculates S1, S2, S3 and S4, ✓ a = 4 ✓ solves simultaneously for b and c. (3)
6(d)(1)	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	✓✓ answer (2)

6(d)(2)	$Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ OR/OF $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6 \quad b + c = -10 \dots\dots\dots(1)$ $Q_2 = 16 + 2b + c = -9 \quad 2b + c = -25 \dots\dots\dots(2)$ $b = -15 \dots\dots\dots(2) - (1)$ $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\,634$	$\checkmark\checkmark -6 + 4(128)^2 - 7(128)$ $\checkmark$ answer (3) $\checkmark a = 4$ $\checkmark Q_n = 4n^2 - 15n + 5$ $\checkmark$ answer (3) [12]
7(a)	No, there will be no intersection between the graphs. Minimum value of $3(x-1)^2 + 5$ is 5 <i>Nee, daar saal geen snyding tussen die grafieke wees nie, Min waarde van $3(x-1)^2 + 5$ is 5</i> OR/ OF $3(x-1)^2 + 5 = 3$ $3(x-1)^2 = -2$ $(x-1)^2 = -\frac{2}{3}$ No, there will be no intersection between the graphs. <i>Nee, daar sal geen snyding tussen die grafieke wees nie.</i>	$\checkmark$ answer $\checkmark$ reason $\checkmark$ answer $\checkmark$ reason [2]
7(b)	$3(x-1)^2 + 5 = 3 + k$	$\checkmark$ answer

	$3(x-1)^2 = k-2$ $k-2 > 0$ $k > 2$ For all real values of x / vir alle reële waardes van x	✓ reason [2]
8(a)	$T_n = a + (n-1)d$ $300 = 18 + (n-1)6$ $300 = 18 + 6n - 6$ $6n = 288$ $n = 48$	✓ $a = 18$ & $d=6$ ✓ Answer ✓ 300 [3]
8(b)	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{48} = \frac{48}{2}[2(18) + (47)6]$ $S_{48} = 7632$	✓ subst ✓ answer [2]
8(c)	Sum of all numbers from 1 to 300 <i>Som van allegetalle van 1 tot 300</i> $S_{300} = \frac{300}{2}[2(1) + (299)(1)]$ $S_{300} = \frac{300(301)}{2}$ $S_{300} = 45150$ Sum of numbers not divisible by 6 / <i>Som van die getalle wat nie deelbaar deur 6 is nie.</i> $= 45150 - (7632 + 6 + 12)$ $= 37500$	✓ subst ✓ answer ✓ $(7632 + 6 + 12)$ ✓ answer [4]
9	$S_5 = \frac{5}{2}(1+5) = 15$ $S_4 = \frac{4}{2}(1+4) = 10$ $\therefore T_5 = 15 - 10 = 5$	✓ 15 ✓ 10 ✓ 5 [3]

SESSION NO: 2

TOPIC: FUNCTIONS AND INVERSE FUNCTIONS

Learner Note: Changing from exponential to logarithmic form in real world problems is the most important concept in this section. This concept is particularly useful in Financial Maths when you are required to solve for n .

SECTION A: TYPICAL EXAM QUESTIONS

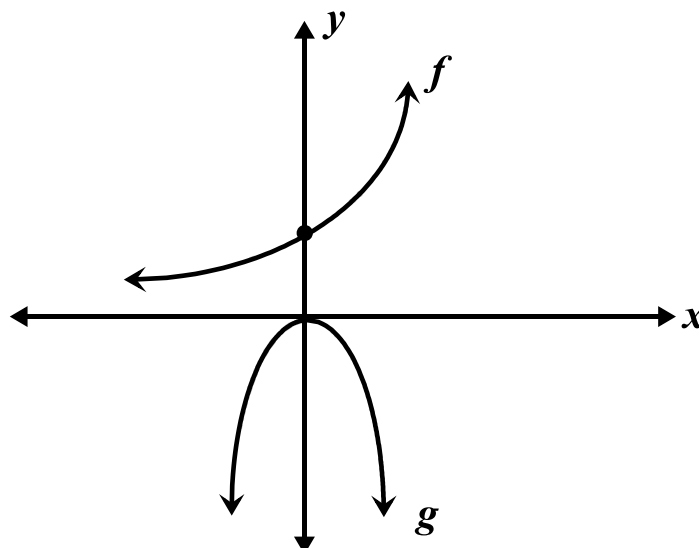
QUESTION 1

Consider the functions: $f(x) = 2x^2$ and $g(x) = \left(\frac{1}{2}\right)^x$

- (a) Restrict the domain of f in one specific way so that the inverse of f will also be a function. (1)
- (b) Hence draw the graph of your new function f and its inverse function f^{-1} on the same set of axes. (2)
- (c) Write the inverse of g in the form $g^{-1}(x) = \dots\dots\dots$ (2)
- (d) Sketch the graph of g^{-1} . (2)
- (e) Determine graphically the values of x for which $\log_{\frac{1}{2}} x < 0$ (1)

QUESTION 2

Sketched below are the graphs of $f(x) = 3^x$ and $g(x) = -x^2$



- (a) Write down the equation of the inverse of the graph of $f(x) = 3^x$ in the form $f^{-1}(x) = \dots$ (2)
- (b) On a set of axes, draw the graph of the inverse of $f(x) = 3^x$ (2)
- (c) Write down the domain of the graph of $f^{-1}(x)$ (1)
- (d) Explain why the inverse of the graph of $g(x) = -x^2$ is not a function. (1)
- (e) Consider the graph of $g(x) = -x^2$
- (1) Write down a possible restriction for the domain of $g(x) = -x^2$ so that the inverse of the graph of g will now be a function. (1)
 - (2) Hence draw the graph of the inverse function in (1) (2)

QUESTION 3

Given: $g(x) = \left(\frac{1}{2}\right)^x$

- (a) Write the inverse of g in the form $g^{-1}(x) = \dots$ (2)
- (b) Sketch the graph of g^{-1} (2)
- (c) Determine graphically the values of x for which $\log_{\frac{1}{2}} x < 0$ (1)

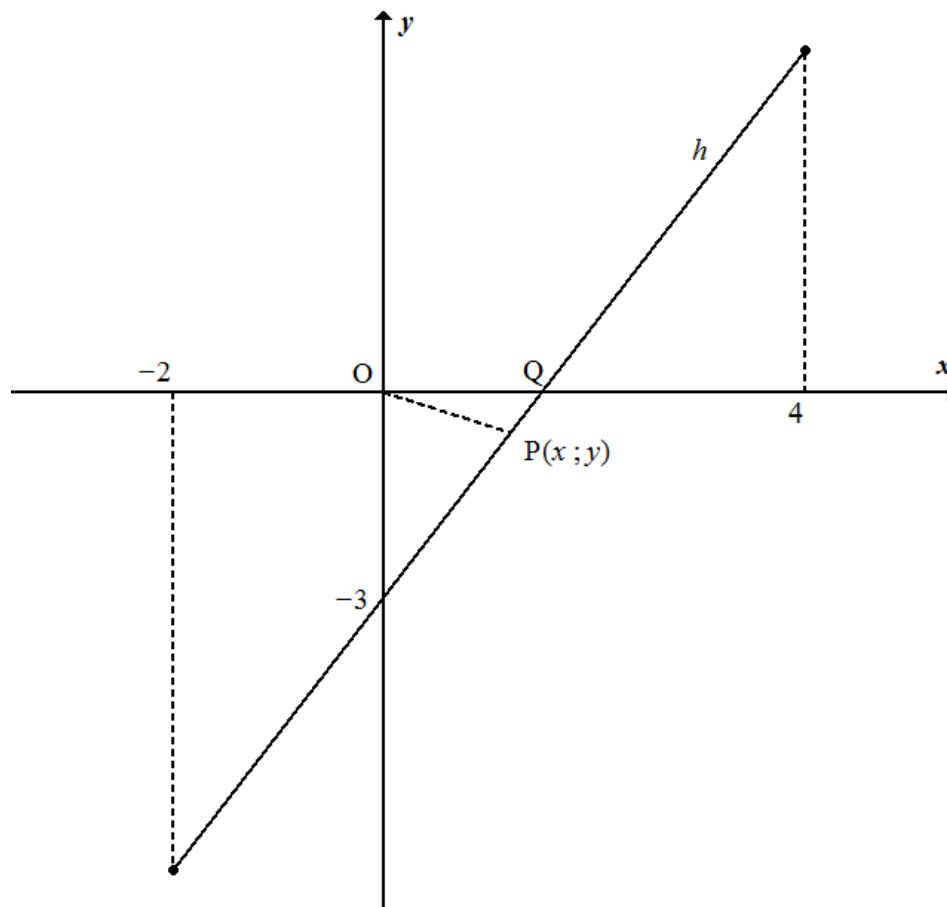
QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

- (a) Write down the equation of the asymptote of f . (1)
- (b) Sketch the graph of f . Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)
- (c) The graph of g is obtained by reflecting the graph of f in the y -axis. Write down the equation of g . (1)

QUESTION 5

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x -intercept of h is Q .



- Determine the coordinates of Q . (2)
- Write down the domain of h^{-1} . (3)
- Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y -intercept and the end points. (3)
- For which value(s) of x will $h(x) = h^{-1}(x)$? (3)
- $P(x; y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP . (5)

QUESTION 6

The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R, x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R, x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any.

(4)

SECTION B: NOTES ON CONTENT

If a number is written in exponential form, then the exponent is called the logarithm of the number. For example, the number 64 can be written in exponential form as $64 = 2^6$. Clearly, the exponent in this example is 6 and the base is 2. We can then say that the logarithm of 64 to base 2 is 6. This can be written as $\log_2 64 = 6$.

The base 2 is written as a sub-script between the “log” and the number 8.

In general, we can rewrite $\text{number} = \text{base}^{\text{exponent}}$ in logarithmic form as follows:

$$\text{number} = \text{base}^{\text{exponent}}$$

$$\therefore \log_{\text{base}}(\text{number}) = \text{exponent}$$

SECTION C: ACTIVITIES
QUESTION 1

A colony of an endangered species originally numbering 1000 was predicted to have a population N after t years given by the equation $N = 1000(0,9)^t$.

- (a) Estimate the population after 1 year. (2)

- (b) Estimate the population after 2 years. (2)
- (c) After how many years will the population decrease to 200? (5)

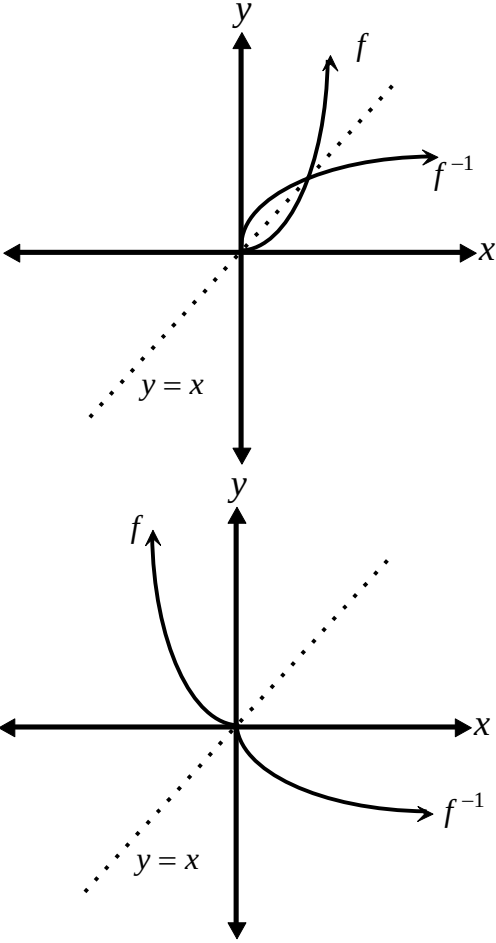
QUESTION 2

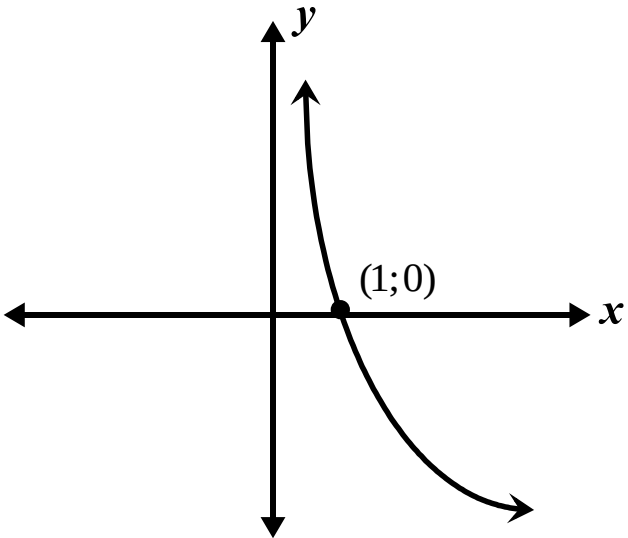
The graph of $f : x \rightarrow \log_a x$ passes through the point (16 ; 2).

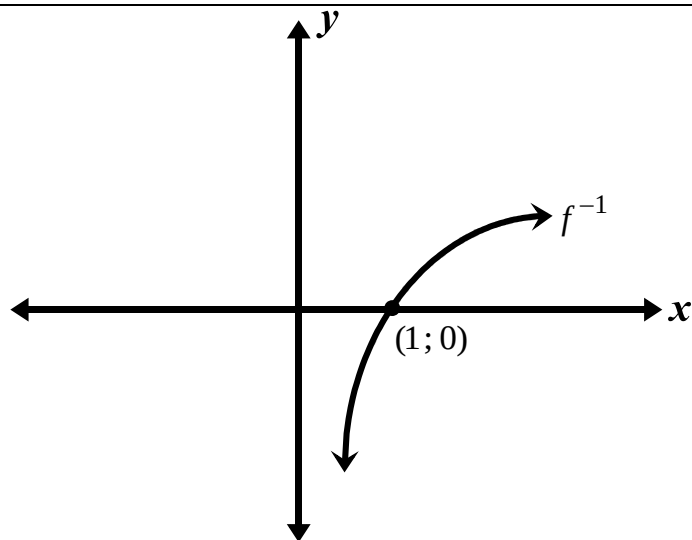
- (a) Calculate the value of a . (3)
- (b) Write down the equation of the inverse in the form $f^{-1}(x) = \dots$ (2)
- (c) Sketch the graphs of f and f^{-1} on the same set of axes. (4)

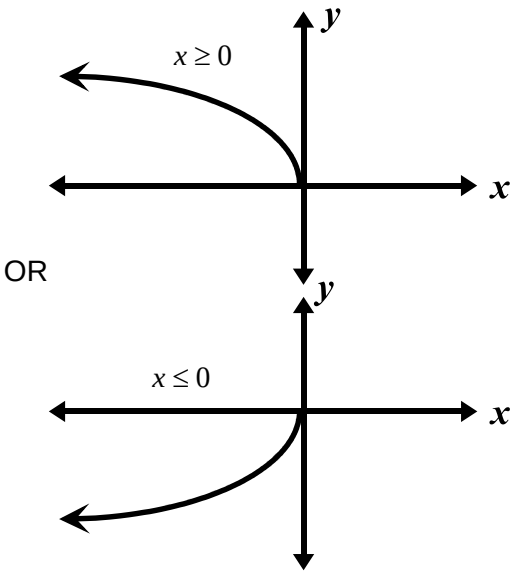
SECTION D: SOLUTIONS FOR SECTION A SESSION 2

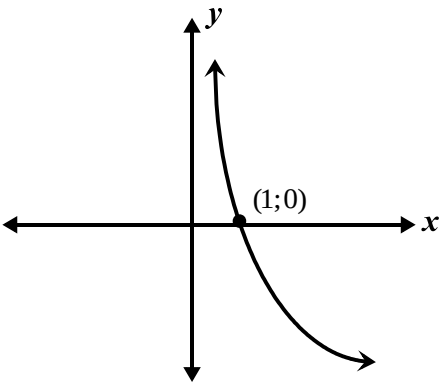
1(a)	$f(x) = 2x^2$ where $x \geq 0$ OR $f(x) = 2x^2$ where $x \leq 0$	$\checkmark x \geq 0$ OR $x \leq 0$ (1)
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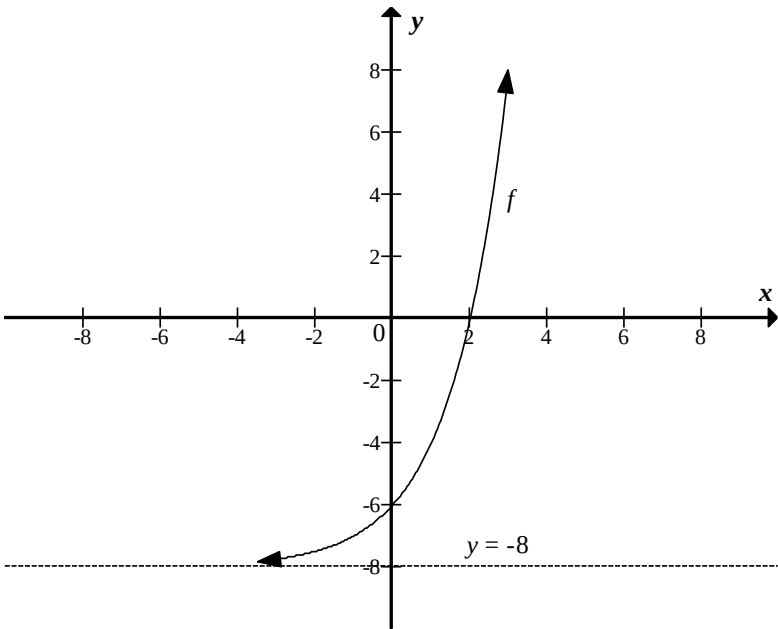
1(b)	 OR	✓ f ✓ f^{-1} (2)
1(c)	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x) = \log_{\frac{1}{2}} x$	✓ $x = \left(\frac{1}{2}\right)^y$ ✓ $g^{-1}(x) = \log_{\frac{1}{2}} x$ (2)

1(d)		✓ shape ✓ (1; 0) (2)
1(e)	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✓ $x > 1$ (1)

2(a)	$y = 3^x$ $\therefore x = 3^y$ $\therefore \log_3 x = y$ $\therefore f^{-1}(x) = \log_3 x$	✓ $x = 3^y$ ✓ $f^{-1}(x) = \log_3 x$ (2)
2(b)		✓ shape ✓ (1; 0) (2)
2(c)	Domain: $x \in (0; \infty)$	✓ $x \in (0; \infty)$ (1)

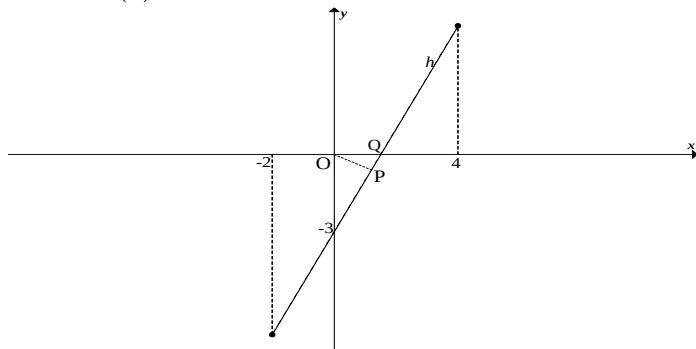
2(d)	The inverse is a one-to-many relation, which is not a function.	✓ one-to-many (1)
2(e)(1)	$x \geq 0$ OR $x \leq 0$	✓ answer (1)
2(e)(2)		✓✓ shape (1)

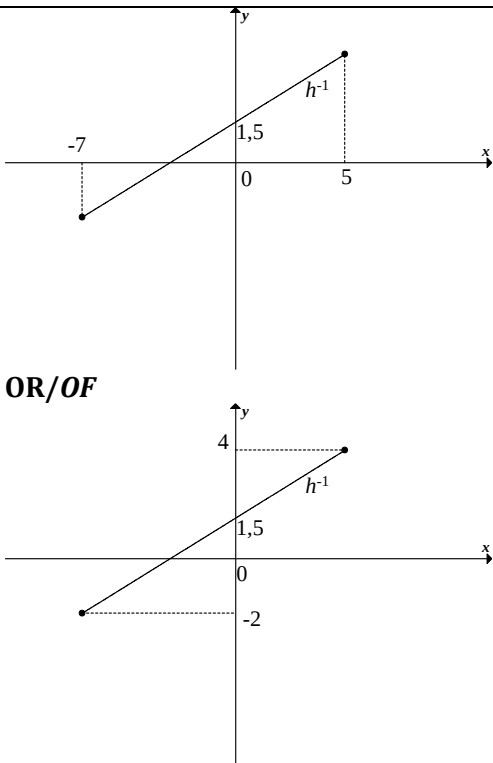
3(a)	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x) = \log_{\frac{1}{2}} x$ Remember that the inverse of a graph is determined by interchanging x and y in the equation of the original graph.	✓ $x = \left(\frac{1}{2}\right)^y$ ✓ $g^{-1}(x) = \log_{\frac{1}{2}} x$ (2)
3(b)	 Don't forget to indicate the coordinates of the intercept with the axes. The y-axis is the asymptote of the graph.	✓ shape ✓ (1; 0) (2)

3(c)	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✓ $x > 1$ (1)
Given: $f(x) = 2^{x+1} - 8$		
4(a)	$y = -8$	✓ $y = -8$ (1)
4(b)		✓ x-intercept ✓ y-intercept ✓ shape ✓ asymptote (only if the graph does not cut the asymptote) (4)
4(c)	$g(x) = 2^{-x+1} - 8$ OR/OF $g(x) = \left(\frac{1}{2}\right)^{x-1} - 8$	✓ answer (1) ✓ answer (1) [6]

QUESTION 5

Given $h(x) = 2x - 3$ for $-2 \leq x \leq 4$.



5(a)	For x-intercepts, $y = 0$ $2x - 3 = 0$ $x = 1,5$ $Q(1,5; 0)$	✓ $x = 1,5$ ✓ $y = 0$ (2)
5(b)	h: $x = -2: y = 2(-2) - 3 = -7$ $x = 4: y = 2(4) - 3 = 5$ Domain of h^{-1}: $-7 \leq x \leq 5$ OR/OF $[-7; 5]$	✓ $h(-2) = -7$ ✓ $h(4) = 5$ ✓ $-7 \leq x \leq 5$ (3)
5(c)	 OR/OF	✓ y-intercept on a straight line ✓ line segment ✓ accurate endpoints (x or y or both) (3)
5(d)	$h(x) = 2x - 3$ For the inverse of h, $x = 2y - 3$ $y = \frac{x+3}{2}$ $h(x) = h^{-1}(x)$ $2x - 3 = \frac{x+3}{2}$ $4x - 6 = x + 3$ $3x = 9$ $x = 3$	✓ $y = \frac{x+3}{2}$ ✓ $2x - 3 = \frac{x+3}{2}$ ✓ $x = 3$ (3)

	OR/OF	
	$h(x) = 2x - 3$ h and h^{-1} intersect when $y = x$ $h(x) = x$ $2x - 3 = x$ $x = 3$ OR/OF $h(x) = 2x - 3$ For the inverse of h , $x = 2y - 3$ $y = \frac{x+3}{2}$ $h^{-1}(x) = x$ $\frac{x+3}{2} = x$ $x+3 = 2x$ $x = 3$	$\checkmark h(x) = x$ $\checkmark 2x - 3 = x$ $\checkmark x = 3$ (3) $\checkmark y = \frac{x+3}{2}$ $\checkmark \frac{x+3}{2} = x$ $\checkmark x = 3$ (3)
5(e)	$OP^2 = (x-0)^2 + (y-0)^2$ $= x^2 + (2x-3)^2$ $= x^2 + 4x^2 - 12x + 9$ $= 5x^2 - 12x + 9$ For OP to be at its minimum, OP^2 has to be a minimum <i>Vir OP om minimum te wees, moet OP^2 'n minimum wees</i> $x = -\frac{b}{2a}$ $= -\frac{-12}{2(5)}$ $\therefore x = \frac{6}{5}$ Minimum length of $OP = \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9} = \sqrt{\frac{9}{5}}$ or $\frac{3}{\sqrt{5}}$ or $1,34$ units OR/OF	$\checkmark OP^2 = x^2 + y^2$ $\checkmark$ substitute $y = 2x - 3$ $\checkmark 5x^2 - 12x + 9$ $\checkmark$ x-value $\checkmark$ answer (5)

	$m_h = 2$ (given) $m_{OP} = \frac{-1}{2}$ $\therefore$ OP has equation $y = \frac{-1}{2}x$ $\frac{-1}{2}x = 2x - 3$ $-x = 4x - 6$ $5x = 6$ $x_p = 1,2$ $y_p = -\frac{1}{2}(1,2) = -0,6$ $OP = \sqrt{(1,2 - 0)^2 + (-0,6 - 0)^2}$ $= 1,34$ or $\sqrt{1,8}$ units OR/OF $O(0;0)$ $P(x; 2x-3)$ $Q\left(\frac{3}{2}; 0\right)$ $OP^2 + PQ^2 = OQ^2$ (pythag) $(x-0)^2 + (2x-3-0)^2 + \left(x-\frac{3}{2}\right)^2 + (2x-3-0)^2 = \left(\frac{3}{2}\right)^2$ $x^2 + 4x^2 - 12x + 9 + x^2 - 3x + \frac{9}{4} + 4x^2 - 12x + 9 = \frac{9}{4}$ $10x^2 - 27x + 18 = 0$ $(5x-6)(2x-3) = 0$ $x = \frac{6}{5} \text{ or } \frac{3}{2}$ Hence, $x = \frac{6}{5}$ at P	$\checkmark m_{OP} = \frac{-1}{2}$ $\checkmark$ equation of OP $\checkmark \frac{-1}{2}x = 2x - 3$ $\checkmark$ x-value $\checkmark$ answer (5) $\checkmark OP^2 = x^2 + y^2$ $\checkmark$ substitute $y = 2x - 3$ $\checkmark 10x^2 - 27x + 18$ $\checkmark$ x-value $\checkmark$ answer (5)
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$$OP^2 = x^2 + (2x - 3)^2$$

$$= \left(\frac{6}{5}\right)^2 + \left(2\left(\frac{6}{5}\right) - 3\right)^2$$

$$= \frac{36}{25} + \frac{9}{25}$$

$$= \frac{9}{5}$$

$$OP = 1,34$$

OR/OF

$$\tan \hat{Q} = 2$$

$$\hat{Q} = 63,43^\circ$$

$$\sin 63,43^\circ = \frac{OP}{1,5}$$

$$OP = 1,34$$

OR/OF

$$OP = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{(x-0)^2 + (2x-3-0)^2}$$

$$= \sqrt{x^2 + 4x^2 - 12x + 9}$$

$$= \sqrt{5x^2 - 12x + 9}$$

By using the chain rule (which is not in the CAPS):

$$\frac{dOP}{dx} = \frac{1}{2}(5x^2 - 12x + 9)^{-\frac{1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}(5x^2 - 12x + 9)^{-\frac{1}{2}} \cdot (10x - 12)$$

$$0 = \frac{1}{2}(10x - 12)$$

$$0 = 5x - 6$$

$$x = \frac{6}{5}$$

$$OP = \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9}$$

$$= 1,34$$

$$\checkmark \tan \hat{Q} = 2$$

$$\checkmark \hat{Q} = 63,43^\circ$$

$$\checkmark \sin 63,43^\circ$$

$$\checkmark \frac{OP}{1,5}$$

$$\checkmark \text{answer}$$

(5)

$$\checkmark OP = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\checkmark \text{substitute}$$

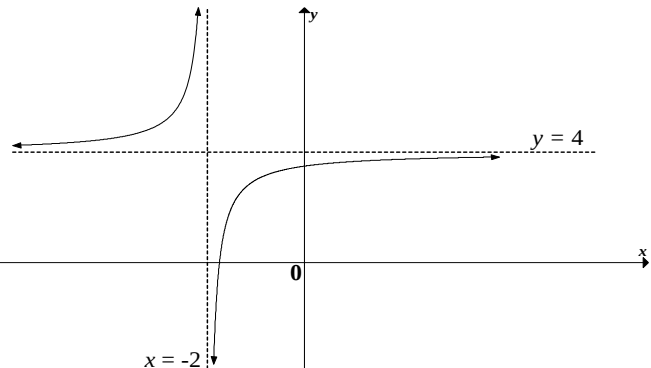
$$y = 2x - 3$$

$$\checkmark 5x^2 - 12x + 9$$

$$\checkmark x\text{-value}$$

$$\checkmark \text{answer}$$

(5)

5(f)(1)	$f'(x) = 2x - 3$ Turning point at $x = \frac{3}{2}$ $f''(x) = 2 > 0$ or $f''\left(\frac{3}{2}\right) > 0$ f has a local minimum at $x = \frac{3}{2}$ <i>f het 'n lokale minimum by $x = \frac{3}{2}$</i> OR/OF $h(x) = f'(x) < 0$ for $x \in (-2 ; 1,5) \Rightarrow f$ is decreasing on the left of Q / <i>f is dalend links van Q.</i> $h(x) = f'(x) > 0$ for $x \in (1,5 ; 4) \Rightarrow f$ is increasing on the right of Q / <i>f is stygend regs van Q.</i> OR/OF $f(x) = x^2 - 3x + c$ f has a minimum value since $a > 0$ <i>f het 'n minimum waarde omdat $a > 0$</i>	✓ Turning point at $x = \frac{3}{2}$ ✓ $f''(x) = 2 > 0$ (2) ✓ decreasing left of Q ✓ increasing right of Q (2) ✓ $f(x) = x^2 - 3x + c$ ✓ explanation (2)
QUESTION 6		
6	If $C(x; y)$ is the centre of the hyperbola/As $C(x; y)$ die middelpunt is van die hiperbool $y = x + 6$ and $x = -2$ $\therefore y = -2 + 6 = 4$ 	✓✓ asymptote $y = 4$ ✓ asymptote $x = -2$ ✓ shape (increasing hyperbolic function) (4) [12]

