SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2019

GRADE 12

SUBJECT: MATHEMATICS

LEARNER ACTIVITIES SOLUTIONS

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SESSION 1

TOPIC: SEQUENCES AND SERIES

SECTION C: ACTIVITIES SOLUTIONS

1(a)	$T_{19} = a + 18d = 11$ $T_{31} = a + 30d = 5$ $\therefore 12d = -6$ $\therefore d = -\frac{1}{2}$ $\therefore a + 18(\frac{-1}{2}) = 11$ $\therefore a = 20$ $\therefore 20;\ 19\frac{1}{2};\ 19...$	✓ $T_{19} = a + 18d = 11$ ✓ $T_{31} = a + 30d = 5$ ✓ $d = -\frac{1}{2}$ ✓ $a = 20$ ✓ sequence (5)
1(b)	$20 + (n-1)\left(-\frac{1}{2}\right) = -29$ $\therefore (n-1)\left(-\frac{1}{2}\right) = -49$ $\therefore n - 1 = 98$ $\therefore n = 99$ $\therefore T_{99} = -29$	✓ Substitution into formula ✓ equating to -29 ✓ $n = 99$ (3)
2(a)	$\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + ... + \frac{180}{181}$ $a = \frac{1}{181}$ $d = \frac{1}{181}$ $n = 180$ $S_{180} = \frac{180}{2}\left[2\left(\frac{1}{181}\right) + (179)\frac{1}{181}\right] = 90[1] = 90$	✓ correct *a* and *d* ✓ correct *n* ✓ S_n formula ✓ correct answer (4)
2(b)	$\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + ... + \left(\frac{1}{181} + \frac{2}{181} + ... + \frac{180}{181}\right)$ $= \frac{1}{2} + 1 + 1\frac{1}{2} + 2 + ... + 90 \quad [a = \frac{1}{2} \quad d = \frac{1}{2} \quad T_n = 90]$ $\therefore \frac{1}{2} + (n-1)\frac{1}{2} = 90$ $\therefore 1 + n - 1 = 180$ $\therefore n = 180$	✓ simplifying fractions to get series ✓ $\frac{1}{2} + (n-1)\frac{1}{2} = 90$ ✓ $n = 180$ ✓ substitution into S_n formula to get 8145 (4)

	$\therefore S_{180}=\frac{180}{2}\left[\frac{1}{2}+90\right]=90\left[90\frac{1}{2}\right]=8145$	

3(a)	$(2p-3)-(1-p)=(p+5)-(2p-3)$ $2p-3-1+p=p+5-2p+3$ $3p-4=-p+8$ $4p=12$ $p=3$	✓ $T_2-T_1=T_3-T_2$ ✓ $2p-3-1+p=p+5-2p+3$ ✓ $p=3$ (3)
3(b)(1)	$T_1=1-(3)=-2$	✓ answer (1)
3(b)(2)	$d=T_2-T_1=(2p-3)-(1-p)$ $\therefore d=(2(3)-3)-(1-3)$ $\therefore d=3-(-2)$ $\therefore d=5$	✓ $(2p-3)-(1-p)$ ✓ $p=3$ (2)
3(c)	The sequence is $-2;3;8;13;18;23;28;33;38;.......$ After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.	✓✓ answer (2)
4	$ar^5=\sqrt{3}$ $ar^7=\sqrt{27}$ $\therefore \frac{ar^7}{ar^5}=\frac{\sqrt{27}}{\sqrt{3}}$ $\therefore r^2=\sqrt{\frac{27}{3}}$ $\therefore r^2=\sqrt{9}$ $\therefore r^2=3$ $\therefore r=\sqrt{3}$ (terms are positive) $\therefore a(\sqrt{3})^5=\sqrt{3}$ $\therefore a=\frac{\sqrt{3}}{(\sqrt{3})^5}$ $\therefore a=\frac{1}{(\sqrt{3})^4}$ $\therefore a=\frac{1}{(3^{\frac{1}{2}})^4}$ $\therefore a=\frac{1}{9}$	✓ $ar^5=\sqrt{3}$ ✓ $ar^7=\sqrt{27}$ ✓ dividing ✓ $r^2=3$ ✓ $r=\sqrt{3}$ ✓ correct working with surds ✓ $a=\frac{1}{9}$ (7)

5(a)	$\sum_{r=1}^{n}(6r-1)=[6(1)-1]+[6(2)-1]+[6(3)-1]+.....+[6(n)-1]=456$ $=5+11+17+....+(6n-1)=456$ $S_n=\frac{n}{2}(2a+(n-1)d)$ $\therefore 456=\frac{n}{2}(2a+(n-1)d)$ $\therefore 456=\frac{n}{2}(2(5)+(n-1)(6))$ $\therefore 456=\frac{n}{2}(10+6n-6)$ $\therefore 456=\frac{n}{2}(4+6n)$ $\therefore 456=2n+3n^2$ $\therefore 0=3n^2+2n-456$ $\therefore (3n+38)(n-12)=0$ $\therefore 3n=-38$ or $n=12$ $\therefore n=-\frac{38}{3}$ or $n=12$ $\therefore n=12$	✓ expanding ✓ correct formula ✓ $456=\frac{n}{2}(2a+(n-1)d)$ ✓ $0=3n^2+2n-456$ ✓ $(3n+38)(n-12)=0$ ✓ $n=-\frac{38}{3}$ or $n=12$ ✓ $\therefore n=12$ (7)
5(b)	$\sum_{k=3}^{n}[(2k-1)n]=5n+7n+9n+...+[(2n-1)n]$ $\therefore a=5n,\ d=2n$ and number of terms $=n-2$ $\therefore S_{n-2}=\frac{n-2}{2}[2a+(n-2-1)d]$ $\therefore S_{n-2}=\frac{n-2}{2}[2(5n)+(n-3)(2n)]$ $\therefore S_{n-2}=\frac{n-2}{2}[10n+2n^2-6n]$ $\therefore S_{n-2}=\frac{n-2}{2}[2n^2+4n]$ $\therefore S_{n-2}=2n(n-2)+n^2(n-2)$ $\therefore S_{n-2}=2n^2-4n+n^3-2n^2=n^3-4n$	✓ expanding ✓ $a=5n,\ d=2n$ ✓ number of terms $=n-2$ ✓ correct formula ✓ substitution ✓ answer (6)
5(c)	$2+5+8+11+14+17$ $d=3$ $T_n=3n-1$ $\sum_{n=1}^{6}(3n-1)$	✓ $d=3$ ✓ $T_n=3n-1$ ✓ $n=1$ ✓ $n=6$ (4)

6(a)	$\sum_{n=1}^{\infty} 2(\frac{1}{2}x)^n$ $= 2(\frac{1}{2}x)^1 + 2(\frac{1}{2}x)^2 + 2(\frac{1}{2}x)^3 + 2(\frac{1}{2}x)^4 + \ldots\ldots\ldots\ldots$ $= x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ The series converges for: $-1 < \frac{1}{2}x < 1$ $\therefore -2 < x < 2$	✓ $r = \frac{1}{2}x$ ✓ $-1 < \frac{1}{2}x < 1$ ✓ $-2 < x < 2$ (3)
6(b)	$a = \frac{1}{2} \quad r = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ $\therefore S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$	✓ a and r ✓ S_∞ formula ✓ $\frac{2}{3}$ (3)

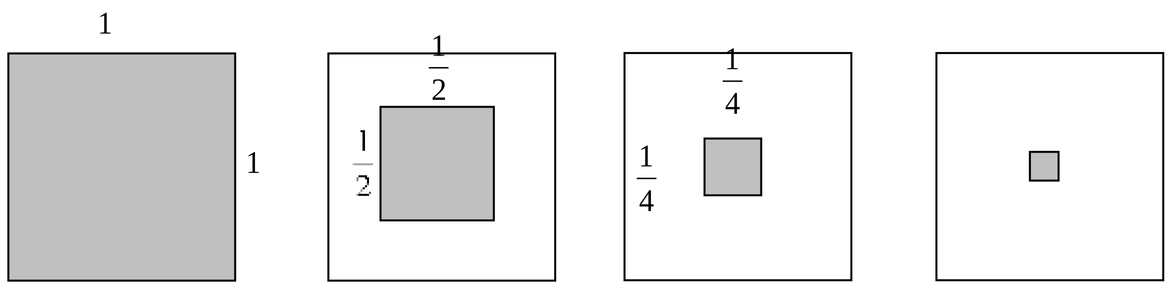


DIAGRAM 1 **DIAGRAM 2** **DIAGRAM 3** **DIAGRAM 4**

7(a)	Area of unshaded square = Area of large square – Area of small shaded square $= (1)(1) - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ $= 1 - \frac{1}{16} = \frac{15}{16}$	✓ $(1)(1) - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ ✓ $\frac{15}{16}$ (2)
7(b)	Sum of the unshaded areas of the first seven squares: $= (1-1) + \left(1-\frac{1}{4}\right) + \left(1-\frac{1}{4^2}\right) + \cdots + \left(1-\frac{1}{4^6}\right)$ $= 7 - \left(1 + \frac{1}{4} + \frac{1}{4^2} + \cdots + \frac{1}{4^6}\right)$	✓ ✓ Getting the pattern for the unshaded areas ✓ correct formula ✓ substitution ✓ answer (5)

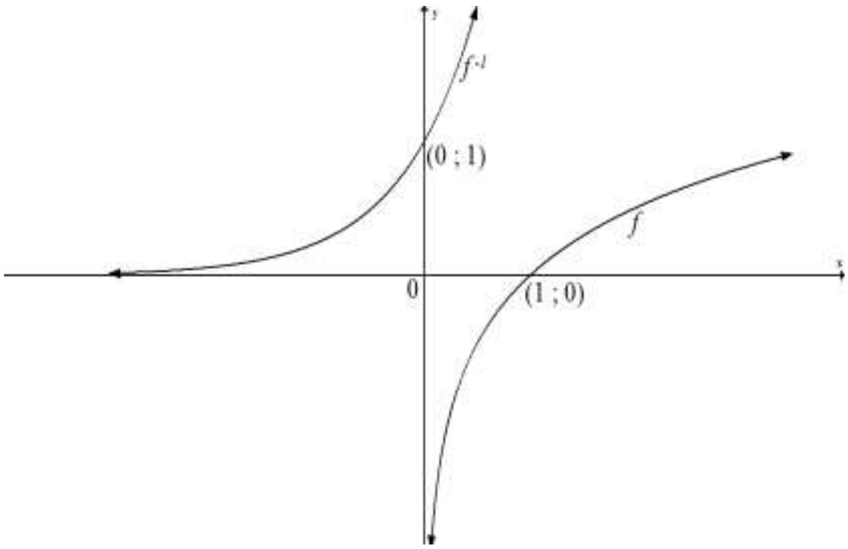
	$=7-\left(\dfrac{1\left(1-\left(\frac{1}{4}\right)^{7}\right)}{1-\frac{1}{4}}\right)$ $=7-1,333251953...$ $=5,67$	
8.	$\therefore S_{\infty}=\dfrac{1,5}{1-\left(\frac{2}{3}\right)}$ $\therefore S_{\infty}=4,5\,\text{m}$ Thus the greatest height is 4,5 m	✓ correct formula ✓ substitution ✓ answer (3)

SESSION 2

TOPIC: FUNCTIONS AND INVERSE FUNCTIONS

SECTION C: ACTIVITIES SOLUTIONS

1(a)	$N=1000(0,9)^{t}$ $=1000(0,9)^{1}$ $=900$	✓$t=1$ ✓900 (2)
1(b)	$N=1000(0,9)^{t}$ $=1000(0,9)^{2}$ $=810$	✓$t=2$ ✓810 (2)
1(c)	$1000(0,9)^{t}=200$ $(0,9)^{t}=\frac{1}{5}$ $t\log(0,9)=\log\frac{1}{5}$ $t=15,27$ After 15,27 years	✓$1000(0,9)^{t}=200$ ✓ $\frac{1}{5}$ ✓ log each side ✓✓ answer (5)

2(a)	$f : x \rightarrow \log_a x$ $y = \log_a x$ $2 = \log_a 16$ $a^2 = 16$ $a = 4$	✓ substitute (16 ; 2) ✓ $a^2 = 16$ ✓ $a = 4$ (3)
2(b)	$y = \log_4 x$ $x = \log_4 y$ $4^x = y$ $f^{-1}(x) = 4^x$	✓ interchange *x* and *y* ✓ $f^{-1}(x) = 4^x$ (2)
2(c)		*f^{-1}* ✓ shape ✓ *y* – intercept *f* ✓ shape ✓ *x* - intercept (4)

SESSION 3

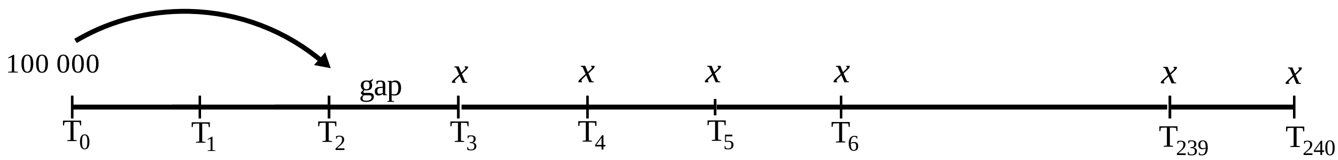
TOPIC: FINANCIAL MATHEMATICS

SECTION C: ACTIVITIES SOLUTIONS

QUESTION 1

(a)	$2\ 500\ 000 = \dfrac{x\left[(1,0075)^{361} - 1)\right]}{0,0075}$ $\therefore \dfrac{2\ 500\ 000 \times 0,0075}{\left[(1,0075)^{361} - 1)\right]} = x$ $\therefore x = \text{R}1354,67$	✓ correct formula ✓ $n = 361$ ✓ $\dfrac{0,09}{12} = 0,0075$ ✓ $\text{F} = 2\ 500\ 000$ ✓ answer (5)
(b)	$2\ 500\ 000 = \dfrac{x\left[1 - \left(1 + \dfrac{0,07}{12}\right)^{-240}\right]}{\left(\dfrac{0,07}{12}\right)}$ $\therefore \dfrac{2\ 500\ 000 \times \left(\dfrac{0,07}{12}\right)}{\left[1 - \left(1 + \dfrac{0,07}{12}\right)^{-240}\right]} = x$ $\therefore x = \text{R}19\ 382,47$	✓ correct formula ✓ $n = 240$ ✓ $\dfrac{0,07}{12}$ ✓ $\text{P} = 2\ 500\ 000$ ✓ answer (5)

QUESTION 2

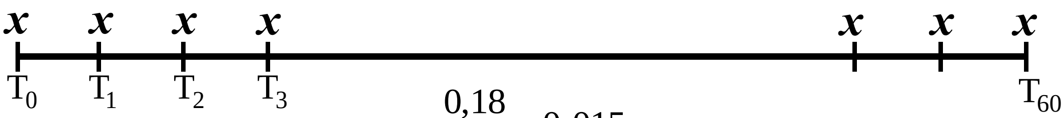


	$\therefore 100\ 000(1,015)^2 = \dfrac{x\left[1 - (1,015)^{-238}\right]}{0,015}$ $\therefore 100\ 000(1,015)^2 \times 0,015 = x\left[1 - (1,015)^{-238}\right]$ $\therefore \dfrac{100\ 000(1,015)^2 \times 0,015}{\left[1 - (1,015)^{-238}\right]} = x$ $\therefore x = \text{R}1591,35$	✓ correct formula ✓ $n = 238$ ✓ $\dfrac{0,18}{12} = 0,015$ ✓ $100\ 000(1,015)^2$ ✓ answer (5)

QUESTION 3

(a)	$A = 200\ 000(1-0,2)^5$ $\therefore A = R65\ 536$	✓ correct formula ✓ answer (2)
(b)	$A = 200\ 000(1+0,16)^5$ $\therefore A = R420\ 068,33$	✓ correct formula ✓ answer (2)
(c)	Sinking fund $= 420\ 068,33 - 65\ 536$ Sinking fund $= 354\ 532,33$	✓ answer (1)

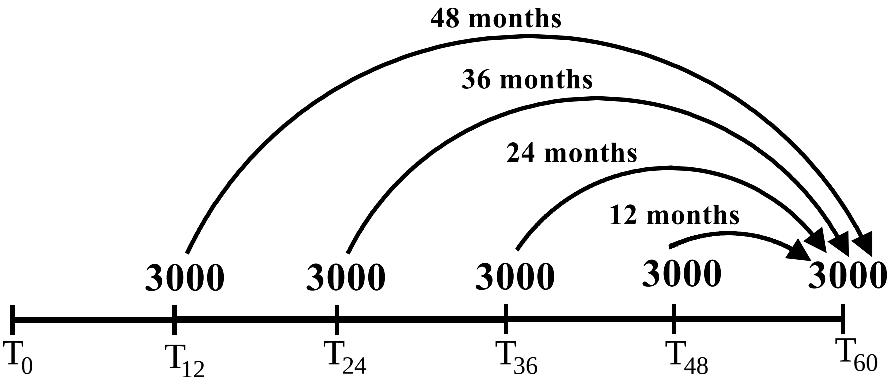
(d) Draw a time line



$$\frac{0,18}{12} = 0,015$$

	$354\ 532,33 = \dfrac{x\left[(1,015)^{61} - 1\right]}{0,015}$ $\dfrac{354\ 532,33 \times 0,015}{\left[(1,015)^{61} - 1\right]} = x$ $\therefore x = R3593,55$	✓ correct formula ✓ $F = 354\ 532,33$ ✓ $n = 61$ ✓ $\dfrac{0,18}{12} = 0,015$ ✓ $x = R3593,55$ (4)

(e)(1) Draw a time line



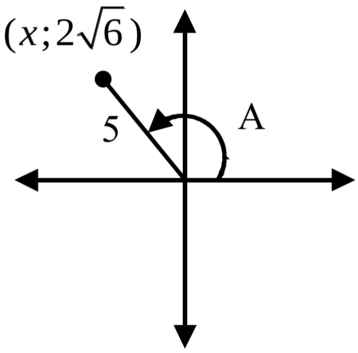
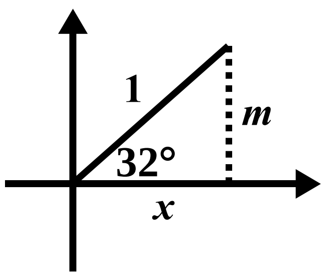
(e)(1)	Future value of the withdrawals: $3000\left(1+\frac{0,18}{12}\right)^{48}+3000\left(1+\frac{0,18}{12}\right)^{36}$ $+3000\left(1+\frac{0,18}{12}\right)^{24}+3000\left(1+\frac{0,18}{12}\right)^{12}+3000$ $=\text{R}22\ 133{,}22$ The reduced value of the sinking fund will be: $\text{R}354\ 532{,}33-\text{R}22\ 133{,}22=\text{R}332\ 399{,}11$	✓ services ✓ $\text{R}22\ 133{,}22$ ✓ reduced value (3)
(e)(2)	If we add R22 133,22 to the original sinking fund amount of R354 532,33, then it will be possible to not only receive the sinking fund amount of R354 532,33 at the end of the five year period, but also be able to make the service withdrawals at the end of each year for the five year period. $354\ 532{,}33+22\ 133{,}22=\frac{x\left[(1{,}015)^{61}-1\right]}{0{,}015}$ $\therefore 376\ 665{,}55=\frac{x\left[(1{,}015)^{61}-1\right]}{0{,}015}$ $\therefore \frac{376\ 665{,}55\times 0{,}015}{\left[(1{,}015)^{61}-1\right]}=x$ $\therefore x=\text{R}3817{,}90$	✓ correct formula ✓ $\text{F}=376\ 665{,}55$ ✓ $n=61$ ✓ $\frac{0{,}18}{12}=0{,}015$ ✓ $x=\text{R}3817{,}90$ (5)
4(a)	$100\ 000=\frac{2000\left[(1{,}08)^{n+1}-1\right]}{0{,}08}$ $\therefore \frac{100\ 000\times 0{,}08}{2000}=(1{,}08)^{n+1}-1$ $\therefore \frac{100\ 000\times 0{,}08}{2000}+1=(1{,}08)^{n+1}$ $\therefore 5=(1{,}08)^{n+1}$ $\therefore \log_{1{,}08}5=n+1$ $\therefore n=19{,}91237188$ $\therefore n\approx 20$ half-years $\therefore n\approx 10$ years (since there are 20 half years in a ten-year period)	✓ correct formula ✓ $\frac{0{,}16}{2}=0{,}08$ ✓ time period $=n+1$ ✓ $5=(1{,}08)^{n+1}$ ✓ $\log_{1{,}08}5=n+1$ ✓ $n=19{,}91237188$ ✓ $n\approx 10$ years (7)

4(b)	$400\ 000 = \dfrac{17000\left[1-(1,04)^{-n}\right]}{0,04}$ $\therefore \dfrac{400\ 000 \times 0,04}{17000} = 1-(1,04)^{-n}$ $\therefore (1,04)^{-n} = 1-\dfrac{400\ 000\times 0,04}{17000}$ $\therefore (1,04)^{-n} = 0,05882352941$ $\therefore 0,05882352941 = (1,04)^{-n}$ $\therefore \log_{1,04} 0,05882352941 = -n$ $\therefore n = 72,23768046$ 18 years 3 months	✓ correct formula ✓ $\dfrac{0,16}{4} = 0,04$ ✓ $0,0588... = (1,04)^{-n}$ ✓ $\log_{0,0625} 1,015 = -n$ ✓ $n = 72,23768046$ ✓ answer (6)

SESSION 4

TOPIC: REVISION OF TRIGONOMETRY

SECTION C: ACTIVITIES SOLUTIONS

1(a)	$\sin A = \dfrac{2\sqrt{6}}{5}$  $x^2 = r^2 - y^2$ $x^2 = 5^2 - \left(2\sqrt{6}\right)^2$ $x^2 = 1$ $x = \pm 1$ $\therefore x = -1$ $\therefore 5\tan A.\cos A = 5\left(\dfrac{2\sqrt{6}}{-1}\right).\left(\dfrac{-1}{5}\right) = 2\sqrt{6}$	✓ $\sin A = \dfrac{2\sqrt{6}}{5}$ ✓ correct quadrant ✓ $x = -1$ ✓✓correct substitution: $5\left(\dfrac{2\sqrt{6}}{-1}\right).\left(\dfrac{-1}{5}\right)$ ✓ correct answer (6)
1(b)(1)	$\sin 32° = \dfrac{m}{1}$  $x^2 + m^2 = 1$ $\therefore x = \sqrt{1-m^2}$ $\sin 328° = \sin 32° = m$	✓ correct quadrant ✓ $x = \sqrt{1-m^2}$ ✓ $\sin 328° = \sin 32°$ ✓ answer (4)

1(b)(2)		✓ $\sin 32°$ ✓ answer (2)
1(b)(3)	$\tan 212°$ $= \tan 32°$ $= \dfrac{m}{\sqrt{1-m^2}}$	✓ $\tan 32°$ ✓ $\dfrac{m}{\sqrt{1-m^2}}$ (2)
2(a)	$\dfrac{\cos 210°.\tan^2 315°}{\sin 300°.\cos 120°}$ $= \dfrac{(-\cos 30°)(-\tan 45°)^2}{(-\sin 60°)(-\cos 60°)}$ $= \dfrac{\left(-\frac{\sqrt{3}}{2}\right)(-1)^2}{\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)}$ $= -2$	✓ $-\cos 30°$ ✓ $(-\tan 45°)^2$ ✓ $-\sin 60°$ ✓ $-\cos 60°$ ✓ evaluating special angles ✓ answer (6)
2(b)	$\dfrac{\sin^2(180°+\theta).\sin(360°-\theta)}{\cos^2(90°-\theta).\cos(90°+\theta)}$ $= \dfrac{(\sin^2\theta)(-\sin\theta)}{(\sin^2\theta)(-\sin\theta)}$ $= 1$	Numerator: ✓ $\sin^2\theta$ ✓ $-\sin\theta$ Denominator: ✓ $\sin^2\theta$ ✓ $-\sin\theta$ ✓ 1 (5)
2(c)	$\cos^2(360°-x)-\sin(180°-x)\cos(90°+x)-\cos^2(180+x)$ $= \cos^2 x-(\sin x)(-\sin x)-\cos^2 x$ $= \sin^2 x$	✓✓✓✓reductions ✓ $\sin^2 x$ (5)

3(a)	$\dfrac{\tan(180°+x)\cos(360°-x)}{\sin(180°-x)\cos(90°+x)+\cos(540°+x)\cos(-x)}$ $=\dfrac{(\tan x)(\cos x)}{(\sin x)(-\sin x)-(\cos x)(\cos x)}$ $=\dfrac{\frac{\sin x}{\cos x}\cos x}{-\sin^2 x-\cos^2 x}$ $=\dfrac{\sin x}{-\left(\sin^2 x+\cos^2 x\right)}$ $=-\sin x$	Numerator: ✓✓ $(\tan x)(\cos x)$ ✓ $\dfrac{\sin x}{\cos x}$ Denominator: ✓✓✓✓ $(\sin x)(-\sin x)-(\cos x)(\cos x)$ ✓ $-\left(\sin^2 x+\cos^2 x\right)$ ✓ $-\sin x$ (9)
3(b)	$\dfrac{\cos(-64°)\tan(-244°)\sin^2 334°}{\cos 566°}$ $=\dfrac{(\cos 64°)(-\tan 244°)(-\sin 26°)^2}{\cos 206°}$ $=\dfrac{(\cos 64°)(-\tan 64°)\sin^2 26°}{-\cos 26°}$ $=\dfrac{(\cos 64°)\left(\frac{-\sin 64°}{\cos 64°}\right)\sin^2 26°}{-\sin 64°}$ $=\sin^2 26°$ $=1-\cos^2 26°$ $=1-p^2$	✓ $\cos 64°$ ✓ $-\tan 64°$ ✓ $\sin^2 26°$ ✓ $-\cos 26°$ ✓ $\dfrac{-\sin 64°}{\cos 64°}$ ✓ $-\sin 64°$ ✓ $1-\cos^2 26°$ ✓ $1-p^2$ (8)
4(a)	$\cos^2 x\left[\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}\right]$ $=\cos^2 x\left[\dfrac{(1+\sin x)+(1-\sin x)}{(1+\sin x)(1-\sin x)}\right]$ $=\cos^2 x\left[\dfrac{2}{1-\sin^2 x}\right]$ $=\cos^2 x\left[\dfrac{2}{\cos^2 x}\right]$ $=2$	✓ $(1+\sin x)+(1-\sin x)$ ✓ $(1+\sin x)(1-\sin x)$ ✓ $\dfrac{2}{1-\sin^2 x}$ ✓ $\dfrac{2}{\cos^2 x}$ ✓ 2 (5)

4(b)	$\frac{1-\cos^2\theta}{\cos^2\theta+2\cos\theta+1}$ $=\frac{(1+\cos\theta)(1-\cos\theta)}{(\cos\theta+1)(\cos\theta+1)}$ $=\frac{1-\cos\theta}{1+\cos\theta}$	✓ $(1+\cos\theta)(1-\cos\theta)$ ✓ $(\cos\theta+1)(\cos\theta+1)$ ✓ $\frac{1-\cos\theta}{1+\cos\theta}$ (3)